

# EFFICIENT FRONTIER ??

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*"Wide diversification is only required when investors do not understand what they are doing."*

- Warren Buffett



## Transaction costs

brokers' commissions and spreads



## Diversification

MVT



## TOO IDEAL

Unrealistic



**How to pick a small  
number of stocks to  
build a profitable  
portfolio ???**



# Cardinality Constrained Portfolio Selection

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**ELITE STREAM GRADUATION THESIS**

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- Models and theories

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01

# Background

Models, theories >>> literature review

# Model

## Markowitz Mean-Variance Portfolio Theory

“Investors always allocate in *all* risky assets available in the market to fully diversify away risks”

- Expected return:

$$E(R_p) = \sum_i w_i E(R_i)$$

where  $R_p$  is the return on the portfolio,  $R_i$  is the return on asset  $i$  and  $w_i$  is weighting of component asset  $i$  (that is, the proportion of asset "i" in the portfolio).

- Portfolio return variance:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$$

where  $\rho_{ij}$  is the correlation coefficient between the returns on assets  $i$  and  $j$ .

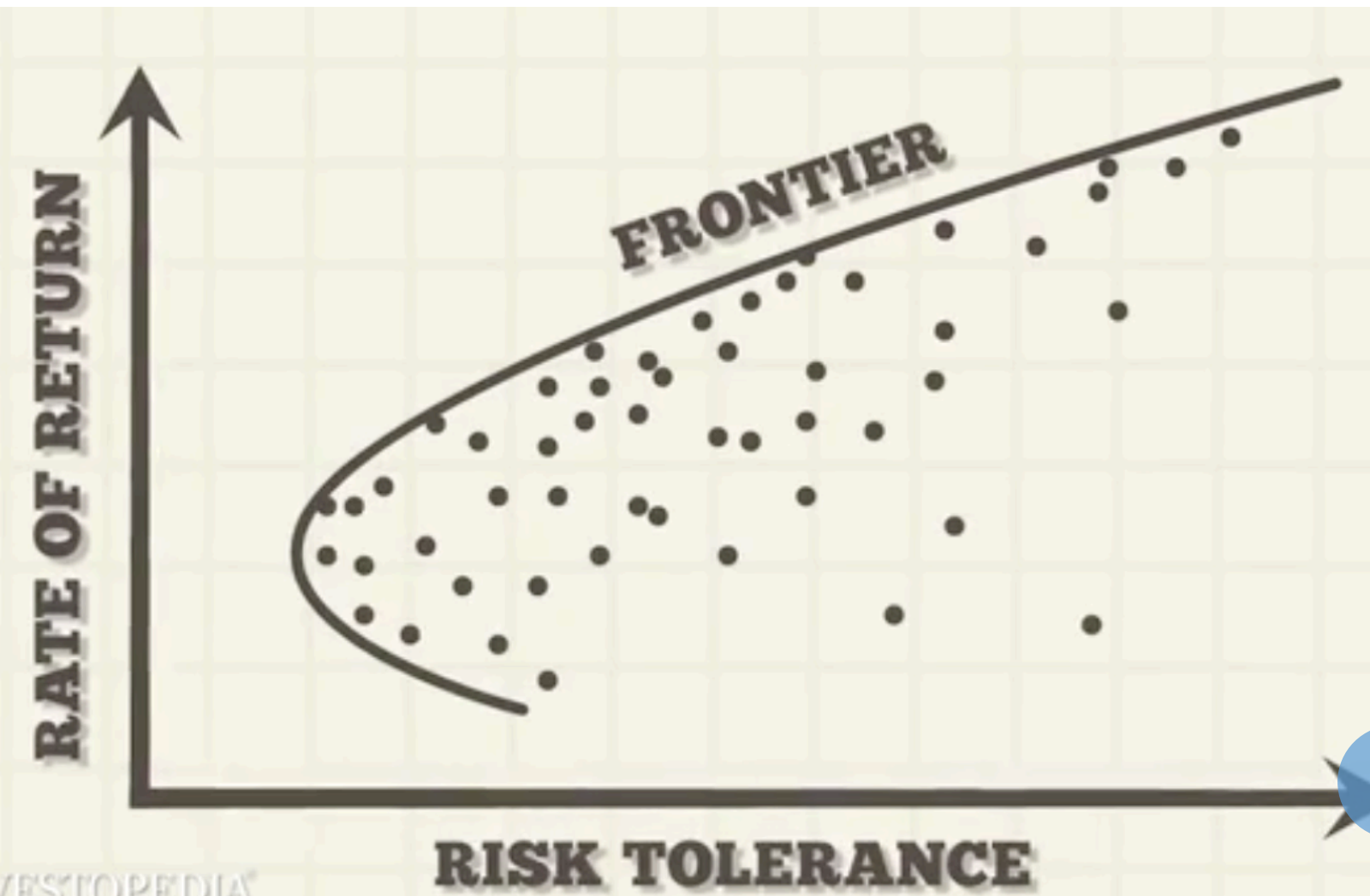
Alternatively the expression can be written as:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij},$$

where  $\rho_{ij} = 1$  for  $i=j$ .







# Model

## Markowitz Mean-Variance Portfolio Theory

“Investors always allocate in *all* risky assets available in the market to fully diversify away risks”



# Problem Formulation

Generalized Markowitz's model

$$\begin{aligned} \min_x & x' Q x \\ \text{s.t.} & \mathbf{r}' x \geq \bar{r} \\ & \mathbf{1}' x = 1 \end{aligned}$$

VS

Cardinality constrained  
mean-variance  
portfolio optimization

$$\begin{aligned} \min_x & x' Q x \\ \text{s.t.} & \mathbf{r}' x \geq \bar{r} \\ & \mathbf{1}' x = 1 \\ & 0 \leq x_i \leq 1 \\ & \sum_{i=1}^n b_i = k \\ & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\ & \text{for } i \text{ from } 1 \text{ to } n \end{aligned}$$

# Existing methodology

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The literature in tackling CCMV in the last two decades can be roughly classified into two categories,

**direct** and **heuristic** algorithms.



## Kening Jiang

Factor Model Based  
Clustering Approach  
for Cardinality  
Constrained Portfolio  
Selection



## Gao & Li

Adopting relaxation  
schemes & invoking  
branch-and-bound  
algorithms  
to attain an optimality.



# Factor Model Based Clustering Approach

Mind flow of **Heuristics**

01

## Transformation

Characterizing risky assets by  
factor model



02

## Grouping

Grouping risky assets using  
clustering algorithm



03

## Selection

Identifying representative(s) from  
individual  
groups & forming the optimal  
portfolio



02

## Detailed Process

Data prep & exploration >>> Heuristics >>> Optimization(direct)

# Data Exploration

## Hang Seng Index Components

Data source: Yahoo Finance

Periodicity: weekly

Since 2000

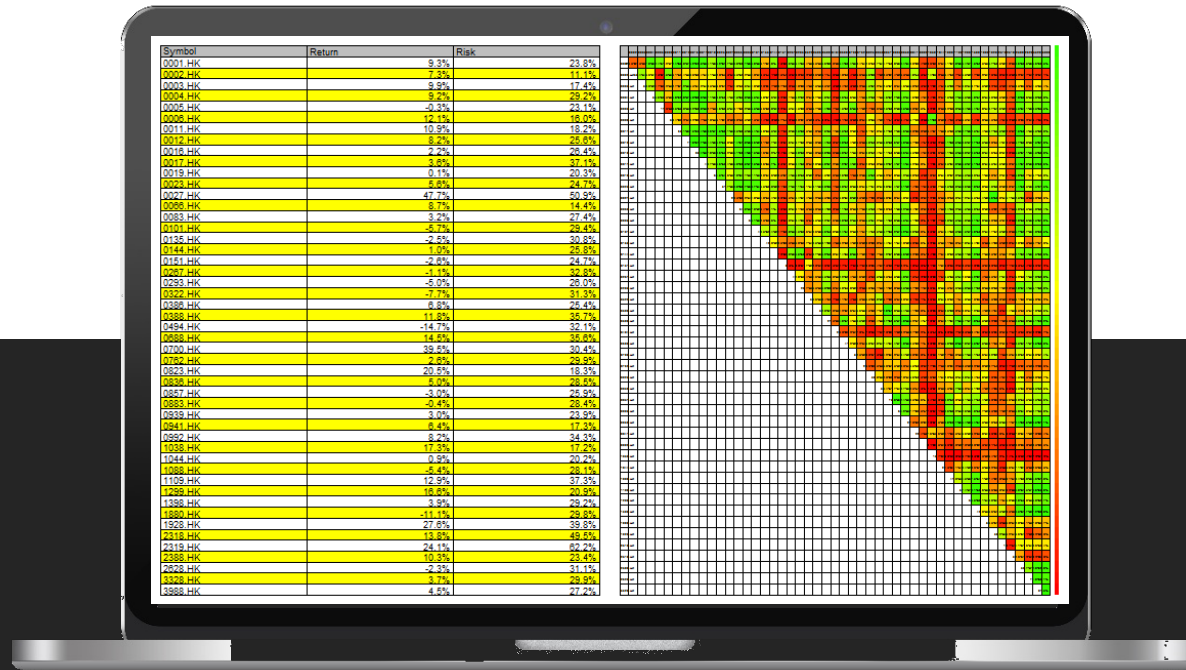
## Risk-free rate

Data source: Bloomberg

Weekly HIBOR rate

## Factors

- Hang Seng Composite Industrial Index
- Fama–French factors



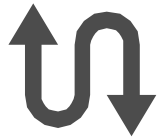




# Step1: Transformation

Characterizing risky assets by factor model

$$r_i = a_i + \sum_{j=1}^m b_{ij} f_j + \epsilon_i, \quad i = 1, 2, \dots, n,$$



## Linear Regression

Find loading coefficients of each risky assets



## multi-Dimensional vectors

each risky asset is projected into multi-Dimensional vectors.

# Step1: Transformation

Characterizing risky assets by factor model

.0

## Correlation

How individuals returns are related to that of other assets



.1

## Industry

How individuals returns are related to industrial sectors



.2

## Fama-French

How individuals returns are related to the market, company size, company Price-to-Book Ratio

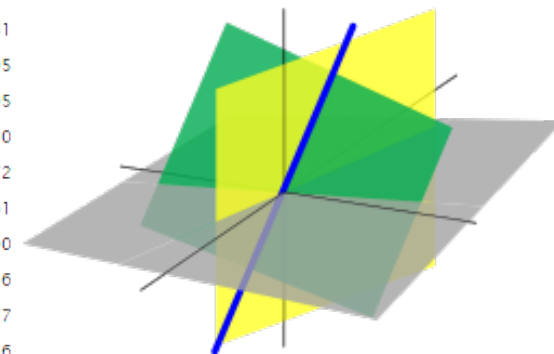
# FM.0: Factor model using covariance matrix

## Factors

Individual stock returns  
49 in total

$$r_i = a_i + \sum_{j=1}^m b_{ij} f_j + \epsilon_i, \quad i = 1, 2, \dots, n,$$

|         | 0001.HK   | 0002.HK   | 0003.HK   | 0004.HK   | 0005.HK    | 0006.HK    | 0011.HK    | 0012.HK   | 0016.HK   | 0017.HK   | 0019.HK   |
|---------|-----------|-----------|-----------|-----------|------------|------------|------------|-----------|-----------|-----------|-----------|
| 0001.HK | 1.0000000 | 0.4206938 | 0.3155842 | 0.5749272 | 0.43654360 | 0.32777751 | 0.55435107 | 0.5875242 | 0.6259690 | 0.5783089 | 0.5369763 |
| 0002.HK | 0.4206938 | 1.0000000 | 0.4602480 | 0.4051255 | 0.27674911 | 0.52577133 | 0.45970681 | 0.3616746 | 0.3995495 | 0.3369651 | 0.3893159 |
| 0003.HK | 0.3155842 | 0.4602480 | 1.0000000 | 0.3478491 | 0.26999847 | 0.39686356 | 0.38860636 | 0.4038291 | 0.3559697 | 0.3261433 | 0.3526009 |
| 0004.HK | 0.5749272 | 0.4051255 | 0.3478491 | 1.0000000 | 0.41618015 | 0.27420512 | 0.54671580 | 0.5780330 | 0.6081915 | 0.5822544 | 0.5167264 |
| 0005.HK | 0.4365436 | 0.2767491 | 0.2699985 | 0.4161801 | 1.0000000  | 0.23154067 | 0.46581221 | 0.3934303 | 0.4356302 | 0.4381724 | 0.4080581 |
| 0006.HK | 0.3277775 | 0.5257713 | 0.3968636 | 0.2742051 | 0.23154067 | 1.0000000  | 0.35495411 | 0.2944021 | 0.2940283 | 0.2617194 | 0.3194495 |
| 0011.HK | 0.5543511 | 0.4597068 | 0.3886064 | 0.5467158 | 0.46581221 | 0.35495411 | 1.0000000  | 0.5000767 | 0.5553767 | 0.5177941 | 0.5266005 |
| 0012.HK | 0.5875242 | 0.3616746 | 0.4038291 | 0.5780330 | 0.39343025 | 0.29440206 | 0.50007668 | 1.0000000 | 0.6475228 | 0.6041713 | 0.4822930 |
| 0016.HK | 0.6259690 | 0.3995495 | 0.3559697 | 0.6081915 | 0.43563022 | 0.29402834 | 0.55537667 | 0.6475228 | 1.0000000 | 0.6662638 | 0.5312272 |
| 0017.HK | 0.5783089 | 0.3369651 | 0.3261433 | 0.5822544 | 0.43817240 | 0.26171936 | 0.51779413 | 0.6041713 | 0.6662638 | 1.0000000 | 0.5050561 |
| 0019.HK | 0.5369763 | 0.3893159 | 0.3526009 | 0.5167264 | 0.40805805 | 0.31944949 | 0.52660051 | 0.4822930 | 0.5312272 | 0.5050561 | 1.0000000 |
| 0023.HK | 0.5431759 | 0.4685364 | 0.3871269 | 0.5360421 | 0.44694398 | 0.34787035 | 0.63906201 | 0.4941708 | 0.5302632 | 0.4927345 | 0.5287356 |
| 0027.HK | 0.4586292 | 0.2616449 | 0.2396220 | 0.4686252 | 0.36572992 | 0.19732505 | 0.44958935 | 0.4191738 | 0.4235740 | 0.4130023 | 0.4091477 |
| 0066.HK | 0.5018967 | 0.4738187 | 0.4050150 | 0.4662514 | 0.37169811 | 0.35924454 | 0.52689361 | 0.4506846 | 0.4709856 | 0.4453303 | 0.4907156 |
| 0083.HK | 0.5943718 | 0.4274930 | 0.3661301 | 0.5876640 | 0.43213608 | 0.31273354 | 0.52749130 | 0.6308389 | 0.6987685 | 0.6642624 | 0.5093953 |
| 0101.HK | 0.5603976 | 0.3938248 | 0.3833570 | 0.5716982 | 0.41602587 | 0.31599528 | 0.55188389 | 0.5612038 | 0.5963671 | 0.5731155 | 0.5050322 |
| 0135.HK | 0.3904098 | 0.2750590 | 0.2411481 | 0.3963267 | 0.39219906 | 0.21562186 | 0.43045563 | 0.3629927 | 0.3988020 | 0.3927398 | 0.3869242 |
| 0144.HK | 0.4456977 | 0.2950564 | 0.2793540 | 0.4467663 | 0.39481150 | 0.25286268 | 0.44906235 | 0.4279906 | 0.4634308 | 0.4656705 | 0.4066230 |



risky asset is projected into **49-D**  
vector spaces.

# Step1: Transformation

Characterizing risky assets by factor model

.0

## Correlation

How individuals returns are related to that of other assets



.1

## Industry

How individuals returns are related to industrial sectors



.2

## Fama-French

How individuals returns are related to the market, company size, company Price-to-Book Ratio



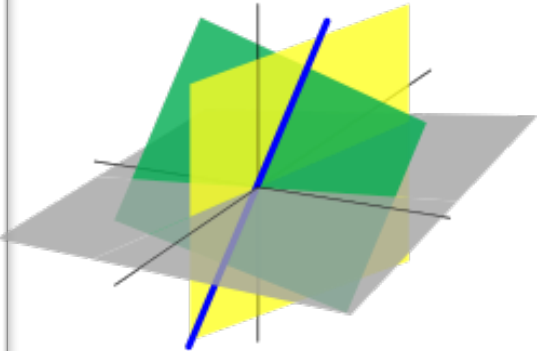
# FM.1: Factor model across industry

## Factors

Hang Seng Composite Index  
11 in total

$$r_i = a_i + \sum_{j=1}^m b_{ij} f_j + \epsilon_i, \quad i = 1, 2, \dots, n,$$

|          | intercept    | HSCIE        | HSCIM        | HSCIIG       | HSCICG       | HSCIS        | HSCIT        | HSCIU        | HSCIF        | HSCIPC       | HSCIIT       | HSCIC        |
|----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| X001_HKW | 0.00280539   | -0.173410575 | 0.027975366  | -0.038763195 | -0.079299651 | 0.076551875  | -0.039381577 | -0.015679225 | 0.170546978  | 0.176970668  | -0.112011009 | 1.033062355  |
| X002_HKW | -0.000512195 | 0.003886528  | -0.052383626 | -0.069401313 | -0.179620575 | -0.000588051 | 0.147554758  | 0.579760105  | 0.051426572  | 0.071097542  | -0.032802855 | 0.149037036  |
| X003_HKW | -0.000913946 | 0.027311403  | -0.121341106 | -0.021809377 | -0.101949764 | 0.020841268  | 0.08418749   | 0.69074844   | 0.156566272  | -0.000257485 | -0.038175704 | 0.135500015  |
| X004_HKW | -0.001077679 | 0.072371848  | -0.041612894 | -0.235016424 | 0.005848378  | 0.155144896  | 0.154930805  | 0.428159702  | -0.175021525 | 0.829197007  | 0.000850592  | -0.050580701 |
| X005_HKW | -0.002165349 | -0.026865938 | -0.119468566 | -0.099563982 | -0.132818517 | 0.129720038  | 0.040510033  | 0.033357557  | 0.835568957  | -0.053864609 | 0.025327504  | 0.036045313  |
| X006_HKW | 0.000264588  | -0.127308818 | 0.053928415  | -0.189238275 | 0.176492695  | -0.148910512 | 0.127777774  | 0.784540523  | 0.019313331  | 0.046294784  | -0.147778222 | 0.340298187  |
| X011_HKW | 0.000418255  | -0.065907216 | -0.159406943 | -0.04531036  | -0.055489332 | 0.090125733  | 0.051561146  | 0.158911776  | 0.484333751  | -0.069161018 | -0.007145648 | 0.348109923  |
| X012_HKW | 0.002315546  | 0.101529071  | -3.95E-05    | -0.068286352 | -0.23088963  | 0.01214863   | 0.096076096  | 0.053247279  | -0.166906172 | 0.87730133   | -0.086277455 | 0.303829344  |
| X016_HKW | -0.001082159 | -0.115134829 | 0.014524403  | -0.17855566  | -0.034320638 | 0.080699478  | 0.00200733   | 0.180919602  | 0.090888855  | 0.688377842  | -0.045767171 | 0.151182955  |
| X017_HKW | -0.002743878 | -0.005007539 | -0.026182425 | -0.178385623 | -0.334940835 | 0.135636034  | 0.064729831  | 0.111691222  | 0.144598366  | 0.890908234  | 0.143014442  | 0.063313869  |
| X019_HKW | -0.000714024 | -0.034961592 | -0.018931781 | -0.099335478 | 0.069661071  | 0.009276619  | 0.109927397  | 0.097474905  | 0.06922989   | 0.163285789  | -0.05683562  | 0.45704398   |
| X023_HKW | 0.00020923   | 0.155509015  | -0.038699893 | -0.19005073  | 0.154279865  | 0.026699002  | 0.019522015  | 0.196472459  | 0.33783242   | 0.158088726  | 0.008658562  | 0.156009548  |
| X027_HKW | 0.003704172  | 0.247654949  | -0.166191105 | -0.104321515 | -0.22116182  | 1.853930606  | -0.275479005 | -0.027593118 | 0.116488061  | -0.130076692 | 0.029610521  | -0.209100451 |
| X066_HKW | 0.000993306  | -0.12096619  | 0.0836649    | 0.005632612  | 0.232749429  | -0.053384215 | 0.113094476  | 0.061230127  | 0.140265091  | -0.0460304   | -0.023773164 | 0.32937143   |
| X083_HKW | -0.001041785 | 0.067646172  | -0.03261529  | -0.234791922 | 0.047745466  | -0.155704697 | -0.010160783 | 0.181421304  | -0.116182979 | 0.98145213   | 0.06494496   | 0.223907989  |
| X101_HKW | -0.002324932 | 0.115442328  | -0.148155514 | -0.180644834 | 0.227606294  | 0.005299311  | 0.123948686  | 0.369111928  | 0.005994226  | 0.552065507  | -0.076957921 | 0.124567786  |
| X135_HKW | -0.003074062 | 0.721408903  | -0.183387656 | 0.242265168  | -0.252479016 | -0.009461757 | -0.093691107 | 0.436556339  | 0.053140155  | 0.148002063  | 0.023957654  | 0.000910765  |
| X144_HKW | -0.000539012 | 0.017505666  | -0.019060913 | 0.002335322  | 0.500358023  | -0.08478226  | 0.193958515  | 0.162615762  | 0.493874332  | -0.219994477 | 0.142229359  | 0.074432826  |
| X151_HKW | -0.001919551 | -0.337903476 | 0.018704223  | -0.64663018  | 2.061619898  | -0.169958186 | 0.047963846  | 0.382459565  | 0.336851667  | -0.215702412 | -0.292564641 | 0.012759639  |
| X267_HKW | 0.001977176  | 0.075923176  | 0.360423745  | 0.0707516    | -0.059130446 | -0.129196882 | 0.036486436  | -0.056341799 | 0.115953529  | -0.022709251 | -0.138649063 | 0.707437872  |
| X293_HKW | -0.003257417 | -0.394121973 | -0.024827331 | 0.014633377  | 0.138821983  | 0.041367212  | -0.03737648  | 0.090654413  | 0.811054635  | -0.059470704 | 0.243853235  | 0.265250772  |
| X322_HKW | -0.001635953 | 0.136026078  | 0.054715127  | -0.481131087 | 1.232393759  | 0.032899079  | 0.113639678  | 0.2123268    | 0.418475369  | -0.518207883 | -0.065062436 | -0.05533427  |
| X386_HKW | 0.001661895  | 0.822971454  | 0.063147185  | -0.150307742 | -0.183450354 | -0.034026587 | -0.050232949 | 0.140565544  | 0.264249403  | 0.029560957  | -0.017784287 | 0.224713296  |
| X388_HKW | 0.004265966  | -0.095110358 | 0.580107092  | 0.096934096  | 0.031481669  | -0.036380773 | 0.049813359  | 0.043837855  | 0.825171746  | -0.240660402 | -0.060896563 | 0.019420978  |
| X494_HKW | -0.002009891 | 0.301656481  | -0.034843877 | -0.360042156 | 1.497758126  | -0.181406303 | -0.118909379 | -0.407047207 | -0.470220701 | 0.402793433  | 0.075799193  | -0.012270121 |



risky asset is projected into **12-D**  
vector spaces.

# Step1: Transformation

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Characterizing risky assets by factor model

.0

## Correlation

How individuals returns are related to that of other assets



.1

## Industry

How individuals returns are related to industrial sectors



.2

## Fama-French

How individuals returns are related to the market, company size, company Price-to-Book Ratio, market momentum

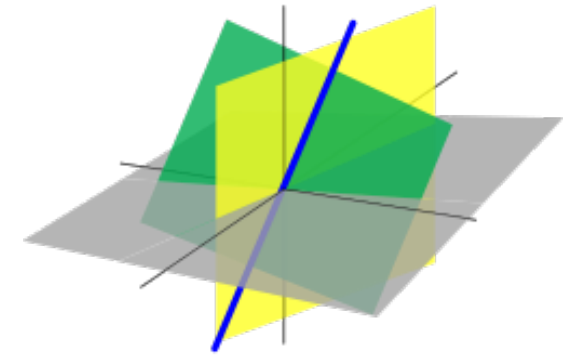
# FM.2 Fama–French three-factor model including momentum

## Factors

SMB  
HML  
Excess market return  
WML  
4 in total

$$r_i = a_i + \sum_{j=1}^m b_{ij} f_j + \epsilon_i, \quad i = 1, 2, \dots, n,$$

|         | alpha    | Mkt.RF   | SMB      | HML         | RF       | WML      |
|---------|----------|----------|----------|-------------|----------|----------|
| 0001.HK | -0.00756 | 1.205127 | -0.31902 | 0.871204233 | 0.041904 | 0.678549 |
| 0002.HK | 0.0026   | 0.132299 | -0.06166 | 0.123003559 | 0.136637 | 0.044677 |
| 0003.HK | 0.001653 | 0.437313 | -0.03272 | 0.456028277 | 0.049889 | 0.203604 |
| 0004.HK | -0.0026  | 1.285279 | -0.49307 | 1.104412822 | -0.26856 | 0.634605 |
| 0005.HK | -0.00554 | 0.774743 | 0.10272  | 0.363527833 | -0.2419  | 0.44955  |
| 0006.HK | 0.004098 | 0.147135 | -0.28245 | -0.03864989 | 0.153335 | 0.070674 |
| 0011.HK | -0.00174 | 0.736789 | -0.1599  | 0.512768    | -0.08815 | 0.484398 |
| 0012.HK | -0.0093  | 1.251505 | -0.31561 | 1.436122797 | -0.14629 | 0.651374 |
| 0016.HK | -0.01068 | 1.28951  | -0.52289 | 1.305734744 | -0.04134 | 0.698076 |
| 0017.HK | -0.01185 | 1.802074 | -0.09975 | 1.97627423  | -0.44935 | 0.689231 |
| 0019.HK | -0.0017  | 0.895887 | -0.20153 | 0.775346795 | -0.27174 | 0.594689 |
| 0023.HK | -0.00391 | 1.140767 | -0.00026 | 0.555289095 | -0.07553 | 0.576377 |
| 0027.HK | 0.020407 | 1.702097 | 0.554019 | 0.53270429  | 0.057159 | 0.286859 |
| 0066.HK | -0.00024 | 0.785553 | -0.27003 | 0.568420659 | -0.07068 | 0.527629 |
| 0083.HK | -0.00028 | 1.496222 | -0.35717 | 1.303607924 | -0.504   | 0.634673 |
| 0101.HK | -0.00353 | 1.092324 | -0.58576 | 0.969312796 | -0.04577 | 0.479788 |
| 0135.HK | 0.009193 | 1.049426 | 0.594554 | 0.128427486 | 0.030552 | 0.281064 |
| 0144.HK | -0.00043 | 1.103851 | 0.078888 | 0.636227369 | -0.07924 | 0.482266 |
| 0151.HK | 0.006704 | 0.380132 | -0.41709 | -0.24682322 | 0.063381 | 0.111668 |
| 0267.HK | -0.00461 | 1.482569 | 0.541803 | 0.419842175 | -0.38747 | 0.54264  |
| 0293.HK | -0.00527 | 0.919047 | -0.05761 | 0.753528754 | -0.29987 | 0.455923 |
| 0322.HK | 0.009467 | 0.566624 | 0.038794 | 1.120322329 | -0.03132 | 0.149352 |
| 0386.HK | 0.003788 | 0.922779 | 0.066959 | 0.096940594 | 0.242193 | 0.29553  |
| 0388.HK | 0.010742 | 1.387892 | 0.312557 | 0.68694357  | -0.2226  | 0.449939 |
| 0494.HK | 0.007375 | 0.757857 | -0.07292 | 0.103239732 | -0.44447 | 0.241201 |
| 0688.HK | 0.004454 | 1.317748 | -0.02586 | 0.912761737 | 0.283291 | 0.371431 |
| 0700.HK | 0.033682 | 1.134236 | -0.00944 | 0.099316123 | 0.064001 | 0.3441   |
| 0762.HK | -0.00261 | 0.883034 | -0.24682 | 0.138594922 | -0.27761 | 0.25787  |
| 0823.HK | 0.011203 | 0.458018 | -0.52526 | 0.120286081 | 0.407254 | 0.267456 |
| 0836.HK | 0.002574 | 0.844133 | -0.37191 | -0.01934385 | 0.563245 | 0.26373  |
| 0857.HK | 0.006537 | 0.97752  | 0.091177 | -0.13055679 | -0.04187 | 0.330975 |
| 0883.HK | 0.003416 | 1.184808 | -0.03983 | -0.26999045 | 0.108219 | 0.518346 |
| 0939.HK | 0.002591 | 0.982069 | -0.11784 | 0.239646347 | 0.267658 | 0.45879  |
| 0941.HK | 0.001514 | 0.769705 | -0.15433 | -0.17732624 | 0.166166 | 0.262296 |
| 0992.HK | -0.0003  | 1.146634 | 0.831257 | 0.180580318 | -0.32999 | 0.34784  |
| 1038.HK | 0.003466 | 0.260223 | -0.11771 | 0.394956974 | 0.199823 | 0.086662 |
| 1044.HK | 0.012927 | 0.671311 | 0.192481 | 0.356005824 | 0.125677 | 0.225889 |
| 1088.HK | 0.002066 | 1.177754 | 0.004169 | 0.110017043 | 0.038955 | 0.503772 |
| 1109.HK | 0.007422 | 1.403384 | -0.27664 | 1.014148098 | 0.166031 | 0.313031 |
| 1299.HK | 0.010131 | 0.90941  | 0.03344  | 0.090836996 | 0.04901  | 0.569379 |
| 1398.HK | 0.002079 | 1.082645 | -0.31657 | 0.138522573 | 0.071281 | 0.479212 |
| 1880.HK | 0.008544 | 1.021166 | 0.566655 | 0.246354626 | 0.047121 | 0.27502  |
| 1928.HK | 0.029925 | 1.155495 | -0.59515 | -0.77875718 | -0.65509 | 0.404168 |
| 2318.HK | 0.013825 | 1.330427 | -0.71224 | -0.36451132 | -0.13884 | 0.339355 |
| 2319.HK | 0.014596 | 1.149514 | -0.21213 | 0.407845782 | -0.23284 | 0.196905 |
| 2388.HK | -0.00137 | 1.064657 | 0.020469 | 0.583729148 | 0.001764 | 0.588225 |
| 2628.HK | 0.000214 | 1.057428 | -0.23685 | 0.229815226 | 0.292212 | 0.342634 |
| 3328.HK | 0.001197 | 1.233896 | -0.24107 | 0.392376047 | 0.272915 | 0.45148  |
| 3988.HK | -0.00112 | 0.979807 | -0.0806  | 0.294393158 | 0.137534 | 0.472078 |



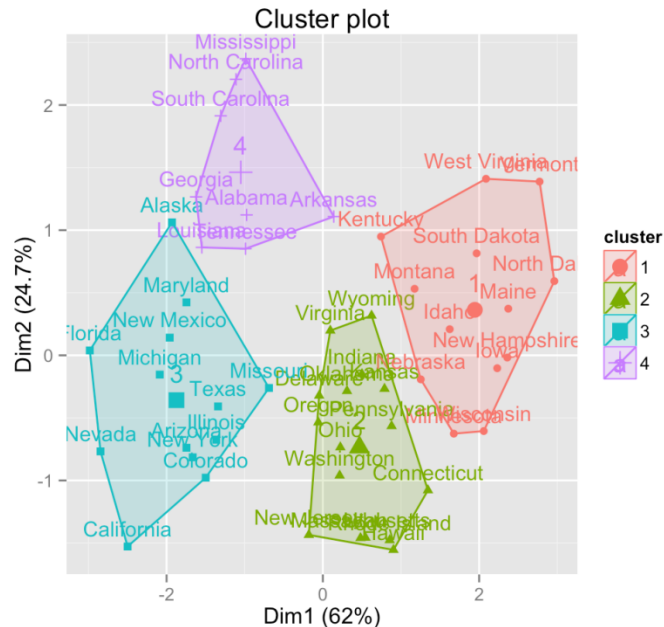
risky asset is projected into **5-D**  
vector spaces.

# Step 2: Grouping risky assets using clustering algorithm

**Goal:** partition all risky assets into  $k$  groups risky assets using clustering algorithm

## Partitioned clustering

Organize elements into disjoint groups

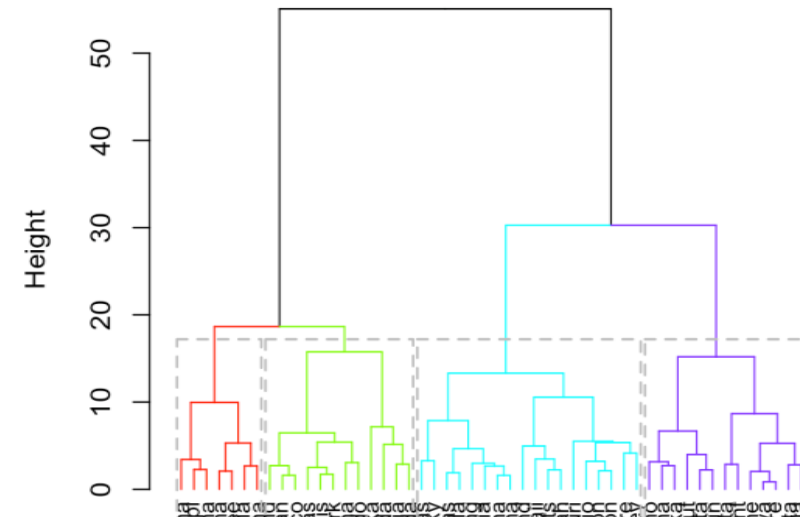


## Hierarchical clustering

Organize elements into trees.

Nested!

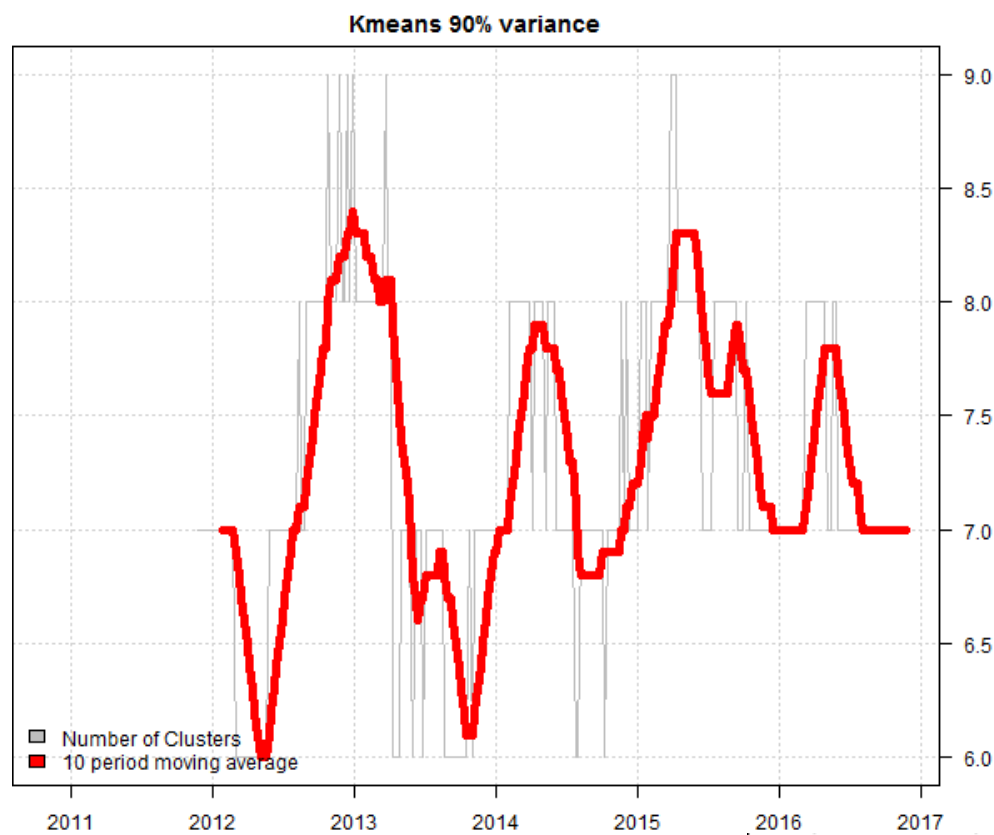
### Cluster Dendrogram



# K-means realization

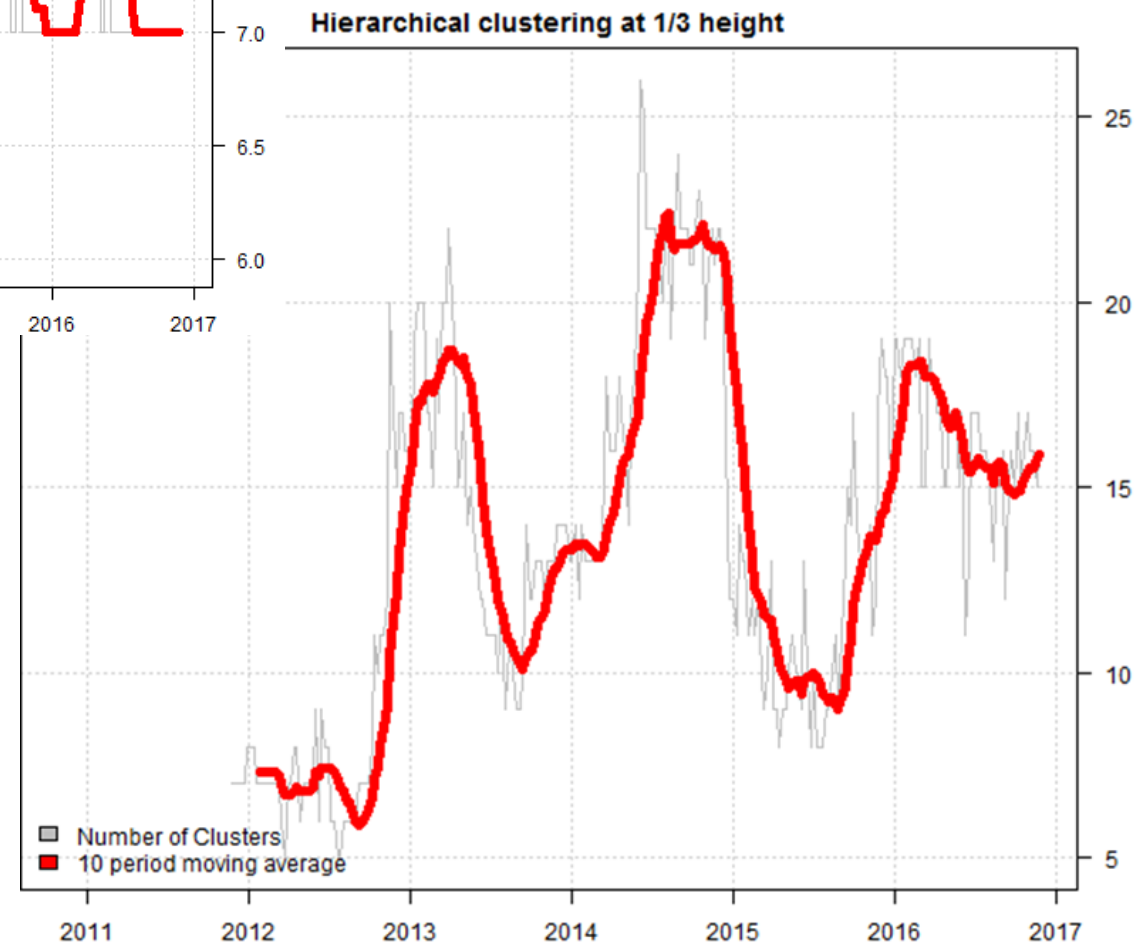
## Preparation

Determine the appropriate number of groups



7

Comparison of number of clusters made by partitioned and hierarchical clustering method



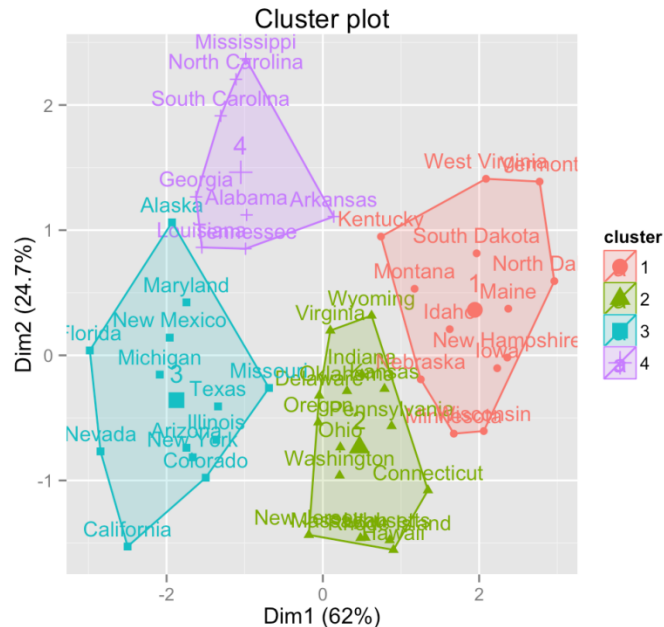


# Step 2: Grouping risky assets using clustering algorithm

**Goal:** partition all risky assets into  $k$  groups risky assets using clustering algorithm

## Partitioned clustering

Organize elements into disjoint groups

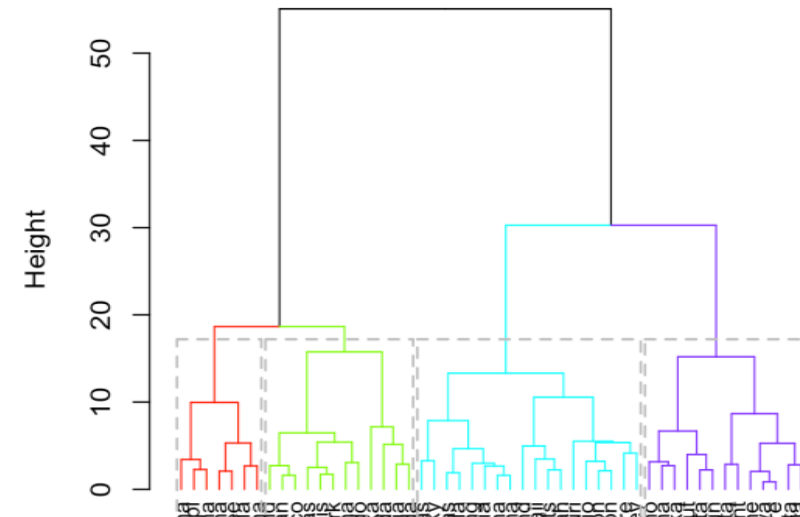


## Hierarchical clustering

Organize elements into trees.

Nested!

### Cluster Dendrogram



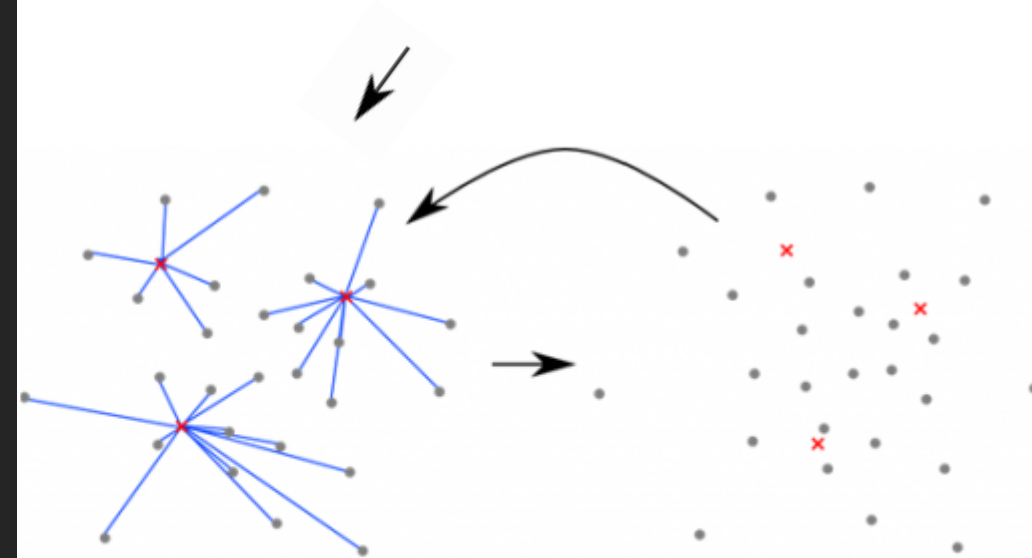
# K-means Clustering

## Algorithm

1. Selects K observations randomly and set them to be the initial centroids
2. Assigns each data point to its closest centroid
3. Recalculates the centroids as the average of all data points in a cluster
4. Assigns data points to their closest centroids
5. Continues steps 3 and 4 until observations are not reassigned or the maximum number of iterations is reached.



1. Initialization



2. Nearest neighbour assignment

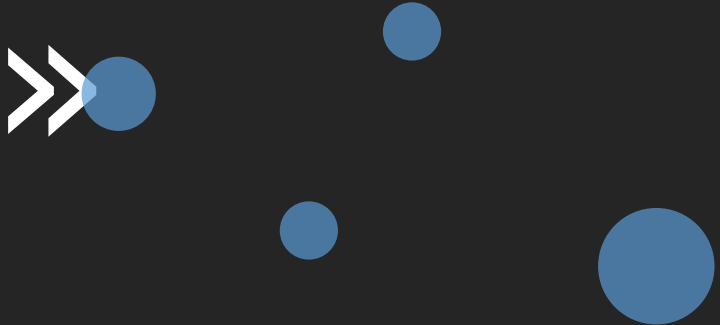
3. Refinement of the means

# Fast Threshold Clustering Algorithm (FTCA)

---

How FTCA creates clusters?

- Pseudo code



*While there are assets that have not been assigned to a cluster*

- ★ If only one asset remaining then
  - ★ Add a new cluster
  - ★ Only member is the remaining asset
- ★ Else
  - ★ Find the asset with the Highest Average Correlation (HC) to all assets not yet been assigned to a Cluster
  - ★ Find the asset with the Lowest Average Correlation (LC) to all assets not yet assigned to a Cluster
  - ★ If Correlation between HC and LC > Threshold
    - ★ Add a new Cluster made of HC and LC
    - ★ Add to Cluster all other assets that have yet been assigned to a Cluster and have an Average Correlation to HC and LC > Threshold
  - ★ Else
    - ★ Add a Cluster made of HC
      - ★ Add to Cluster all other assets that have yet been assigned to a Cluster and have a Correlation to HC > Threshold
    - ★ Add a Cluster made of LC
      - ★ Add to Cluster all other assets that have yet been assigned to a Cluster and have Correlation to LC > Threshold
  - ★ End if
  - ★ End if

*End While*

# Fast Threshold Clustering Algorithm (FTCA)

How FTCA creates clusters?

- Graphic presentation

Assets A through G



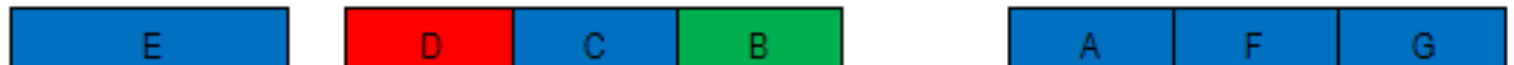
Find asset with both the lowest average correlation (LC) to all other assets and highest/most correlated (MC)



LC

MC

Sort assets > threshold to LC and MC (E and A,F,G), then find the new LC and MC for remaining assets D,C,B



LC

MC

Once the sort is complete, the remaining groups are clusters

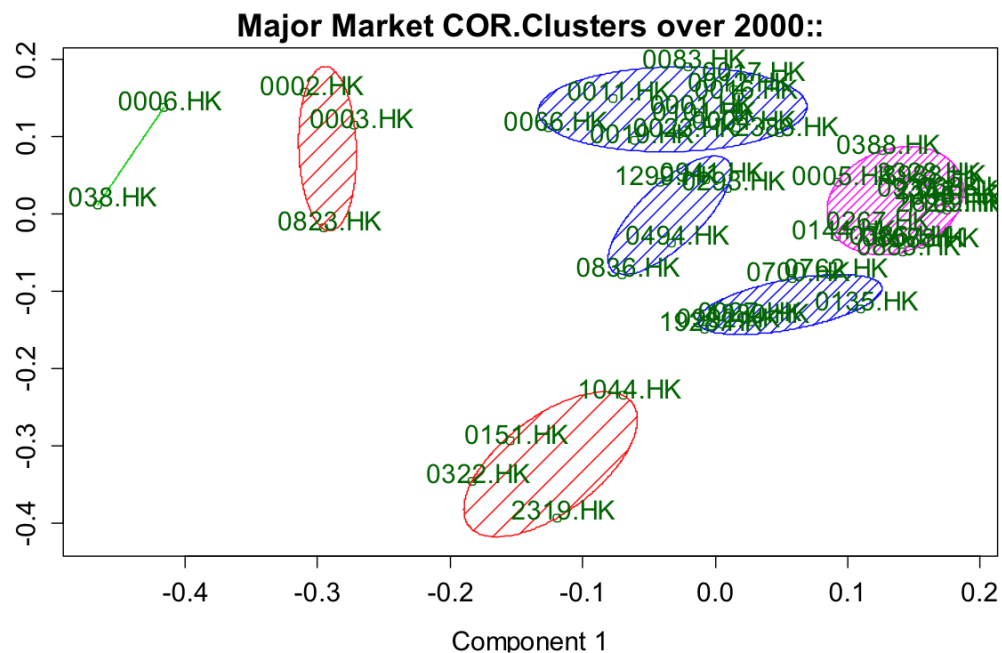


Cluster 1

Cluster 2

Cluster 3

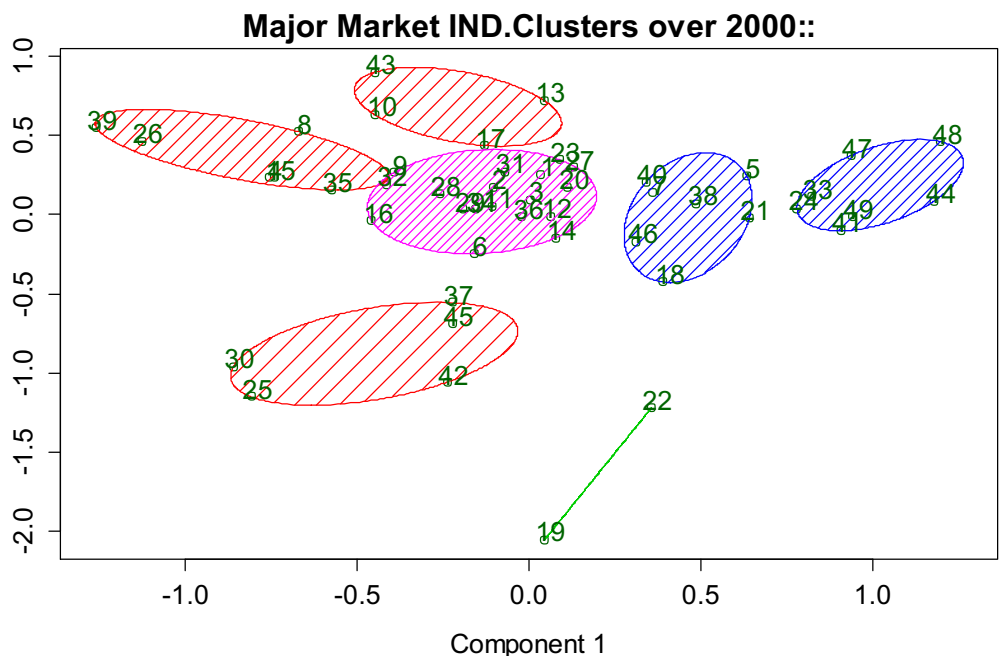
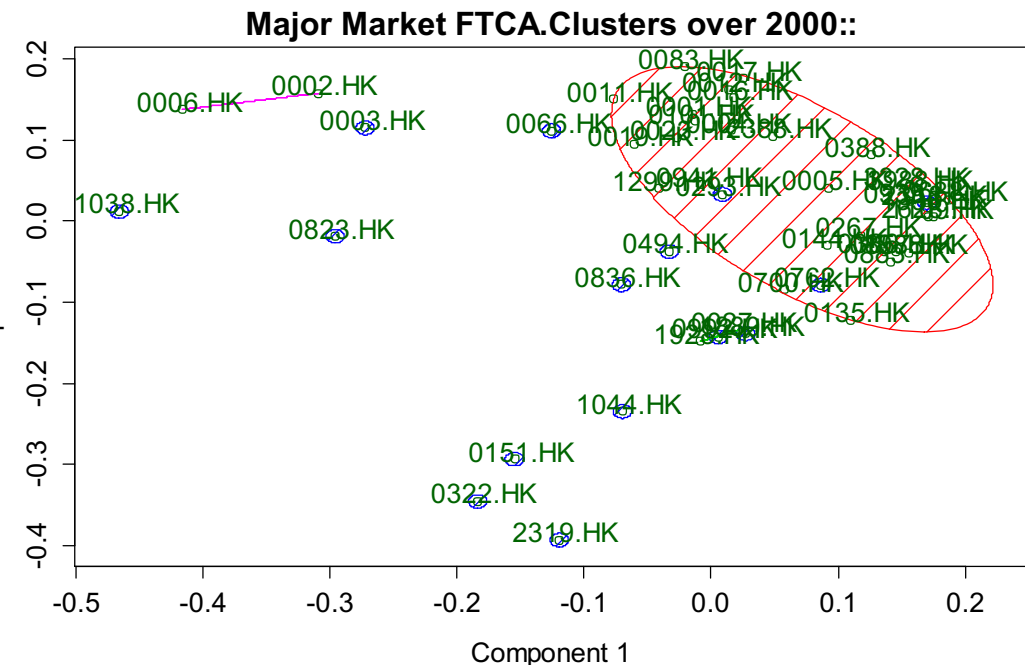
Cluster 4



←k = 7

**COR**

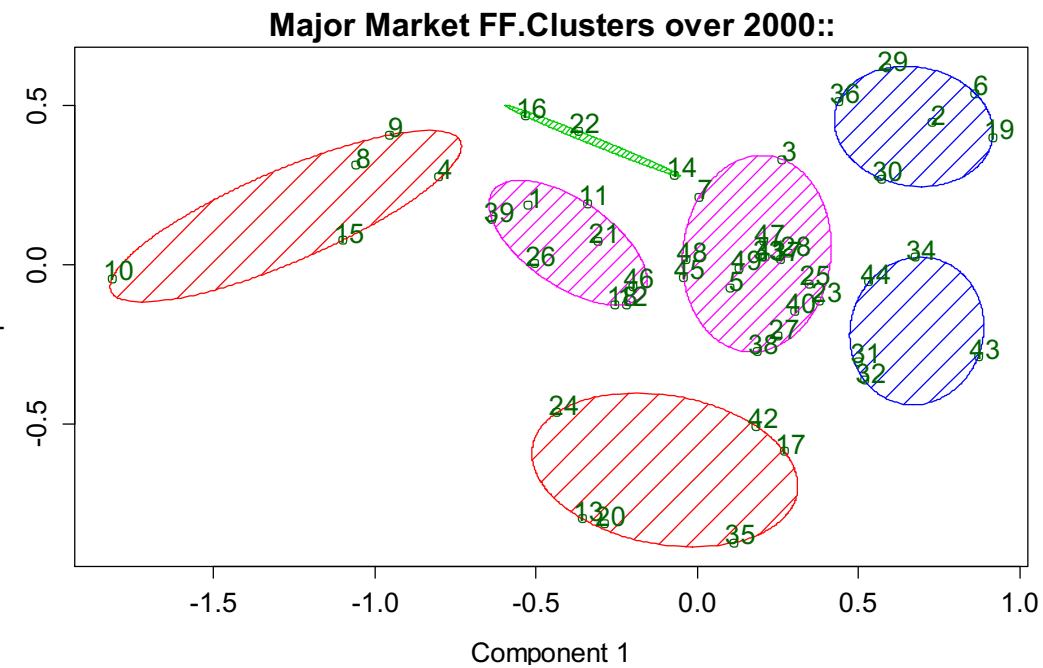
k = 18 →



←IND

k = 7

FF →

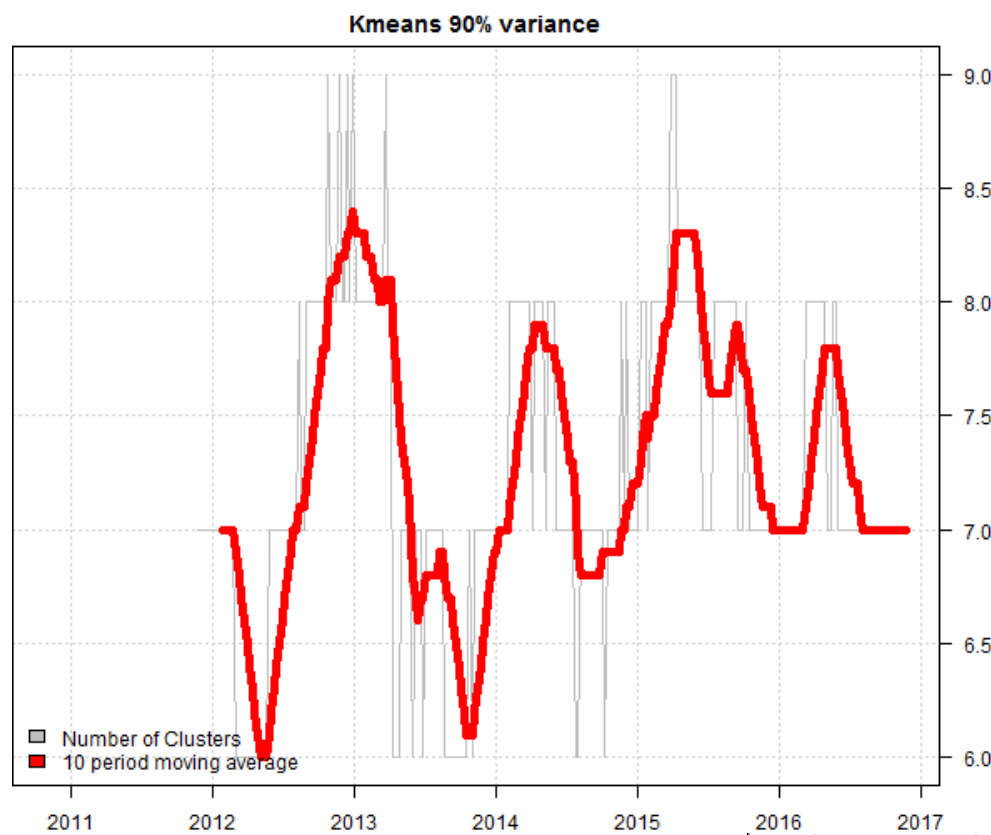




# K-means realization

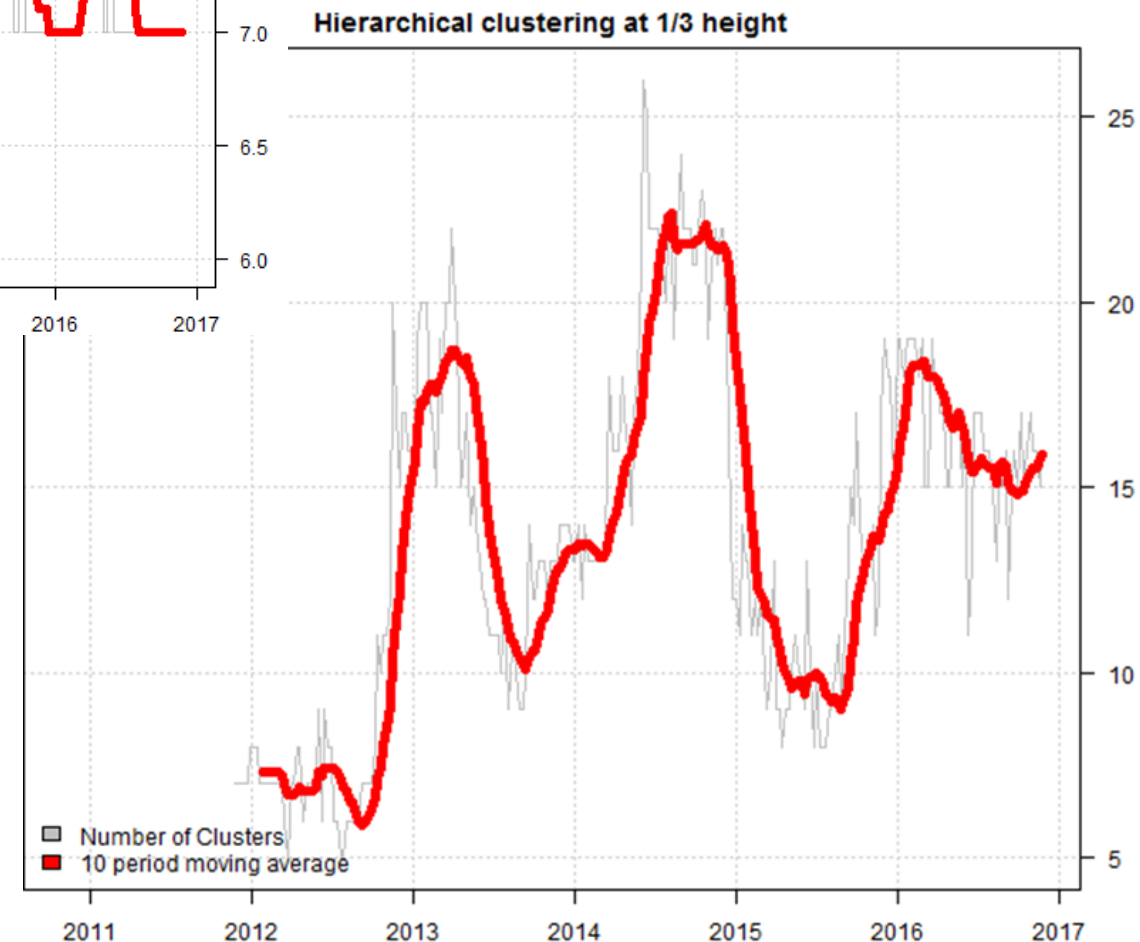
## Preparation

Determine the appropriate number of groups



7

Comparison of number of clusters made by partitioned and hierarchical clustering method



# Step3: Finalize portfolio selection.

---

**Goal:** Identifying representative(s) from groups and forming the portfolio



## Selection by criteria

Concern: best candidate from each group may not necessarily build the best portfolio



## Optimization

Revised and computational feasible problem by adding the clustering result as pre-grouping constraints

# Recall : Problem Formulation

## Generalized Markowitz's model

$$\begin{aligned} \min_x & x' Q x \\ \text{s.t. } & \mathbf{r}' \mathbf{x} \geq \bar{r} \\ & \mathbf{1}' \mathbf{x} = 1 \end{aligned}$$

$$\binom{n}{k} = 18,424$$

VS

$$\prod_{j=1}^k (\beta_j - \alpha_j + 1)$$

## Revised CCMV with pre-grouping

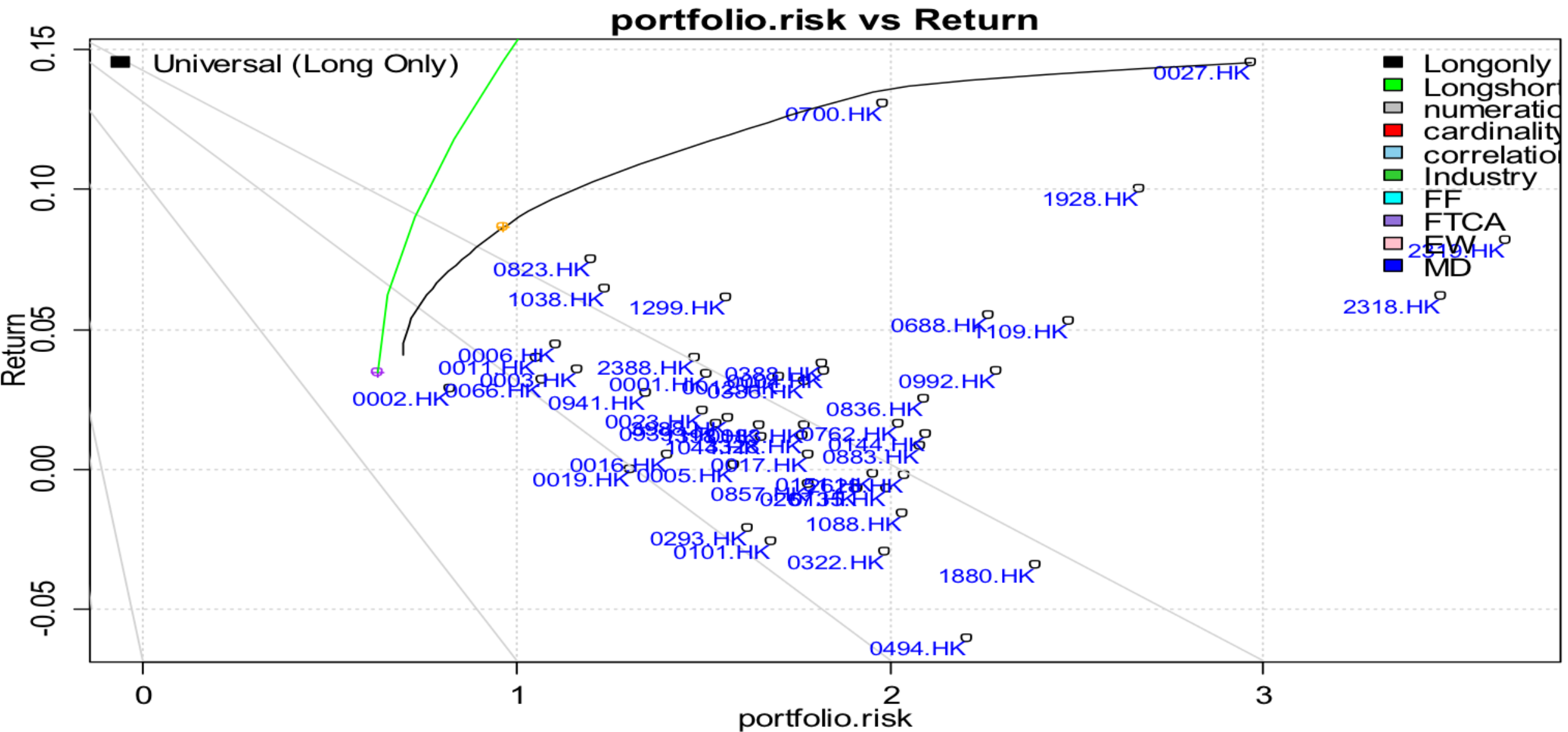
$$\begin{aligned} \min_x & x' Q x \\ \text{s.t. } & \mathbf{r}' \mathbf{x} \geq \bar{r} \\ & \mathbf{1}' \mathbf{x} = 1 \\ & \alpha_j \leq \sum_{i \in I_j}^n b_i \leq \beta_j \\ & \sum_{i=1}^n b_i = k \\ & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\ & i = 1, 2, \dots, n. \\ & j = 1, 2, \dots, k. \end{aligned}$$



03

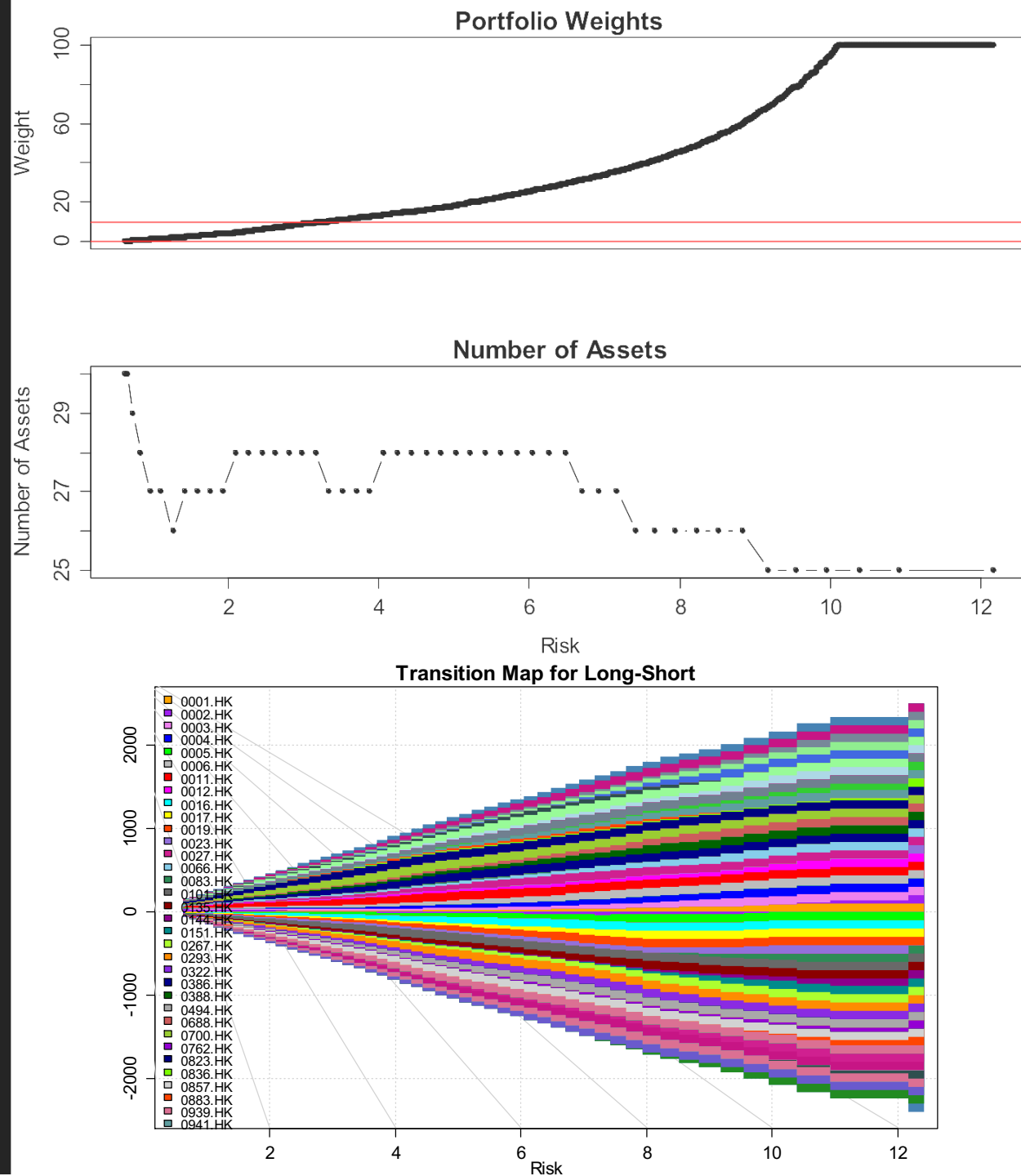
## Outcome

Results and interpretation: Static *EF* >>> *back test*



# Long & short portfolio

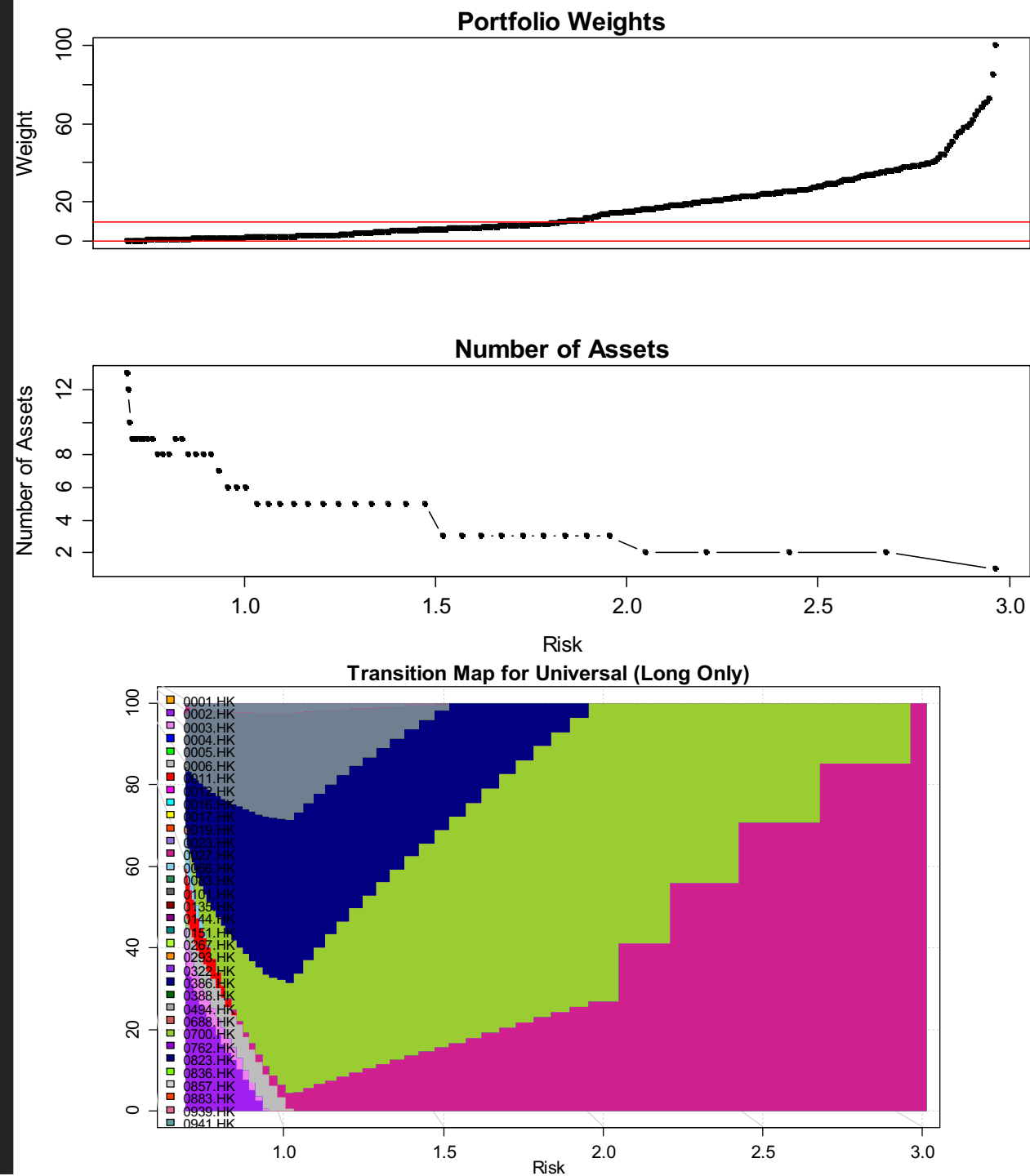
$$\begin{aligned} \min_x & x' Q x \\ \text{s.t. } & \mathbf{r}' x \geq \bar{r} \\ & \mathbf{1}' x = 1 \end{aligned}$$





# Long only portfolio

$$\begin{aligned} \min_x & x' Q x \\ \text{s.t. } & \mathbf{r}' x \geq \bar{r} \\ & \mathbf{1}' x = 1 \\ & 0 \leq x_i \leq 1 \\ & 1 \leq i \leq n \end{aligned}$$



# CCMV portfolio

$$\min_x x'Qx$$

$$\text{s.t. } r'x \geq \bar{r}$$

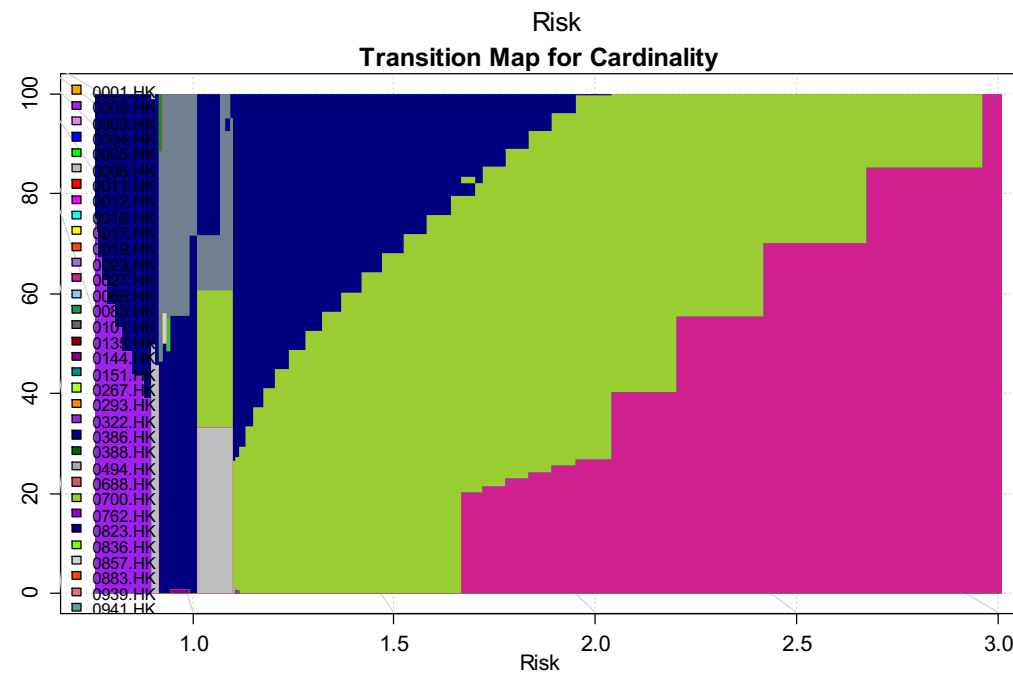
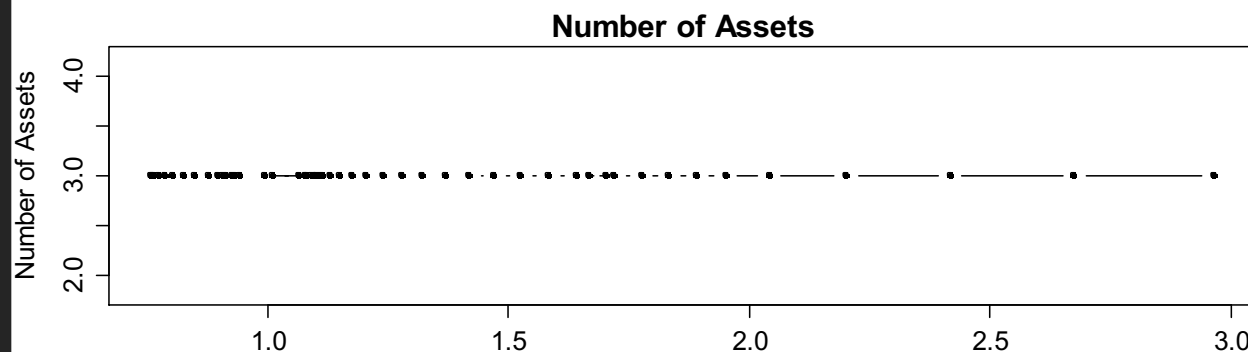
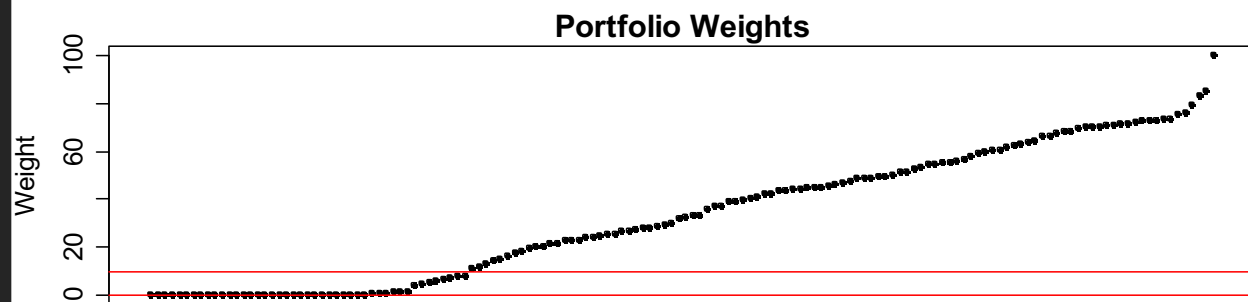
$$\mathbf{1}'x = 1$$

$$0 \leq x_i \leq 1$$

$$\sum_{i=1}^n b_i = k$$

$$b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases}$$

for  $i$  from 1 to  $n$

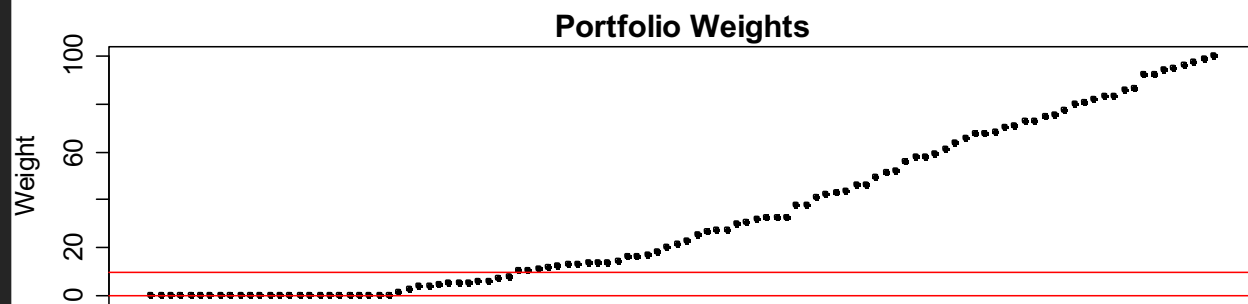


$k = 3$

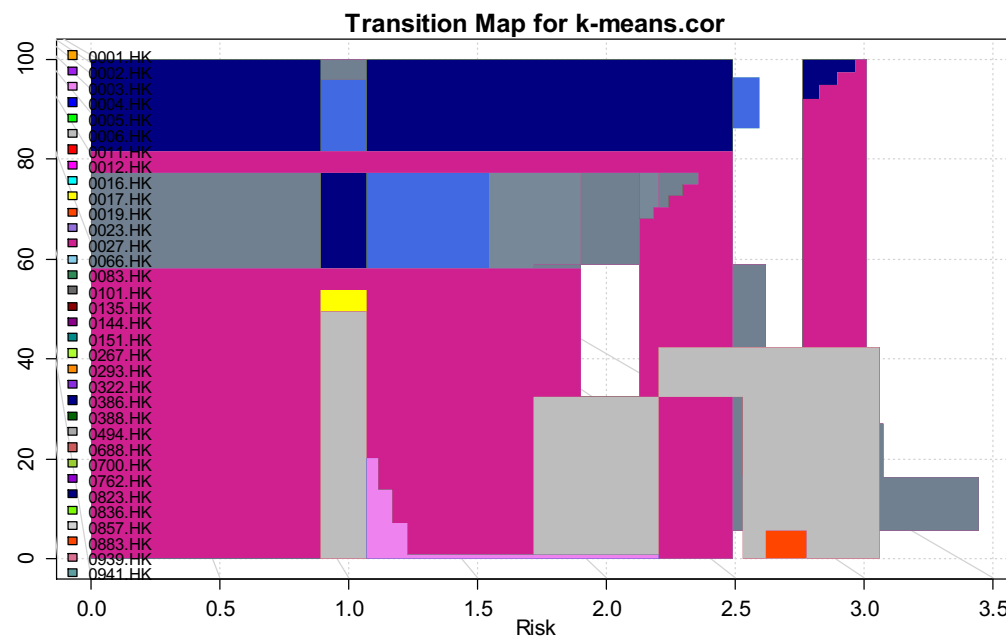
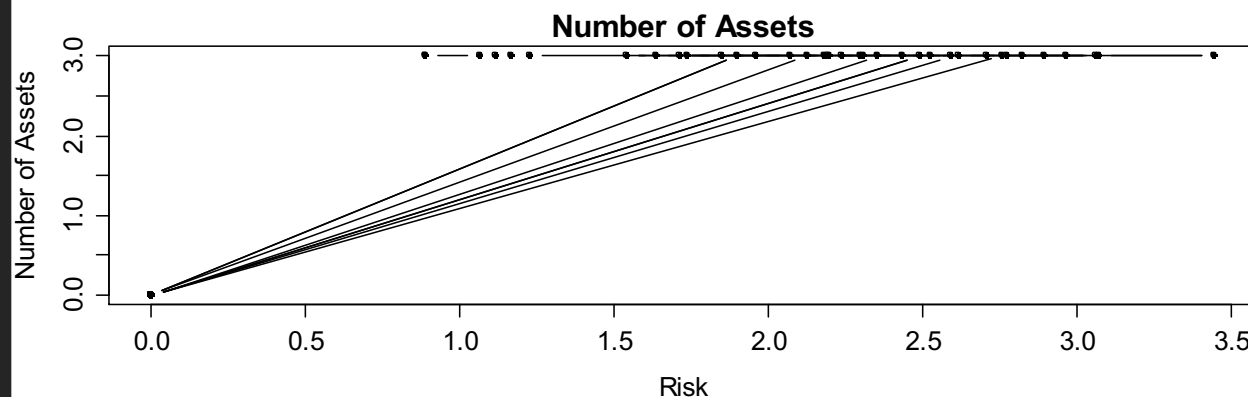


# K-means.cor portfolio

$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j} b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$

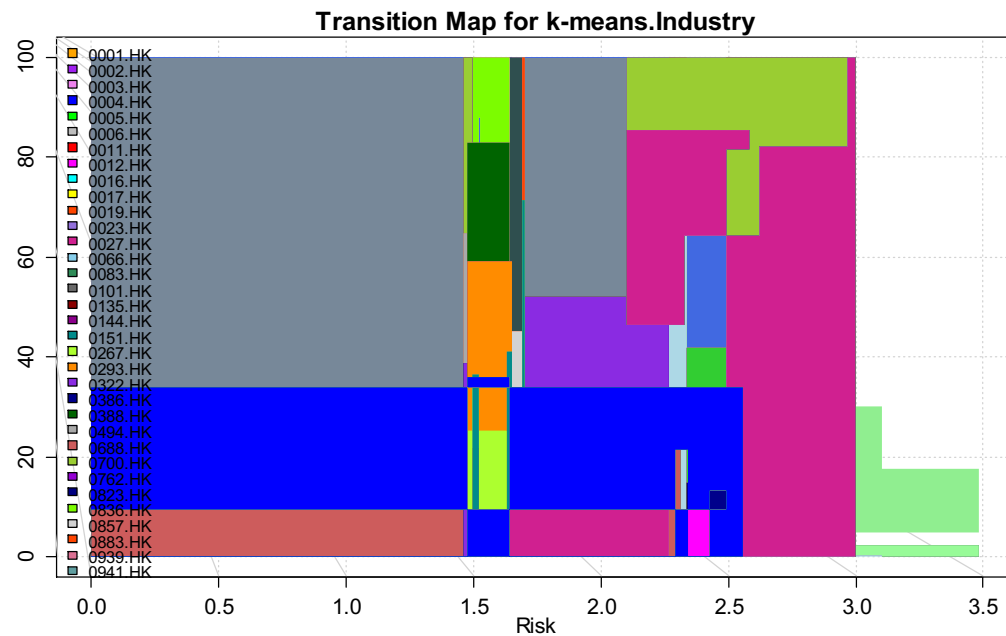
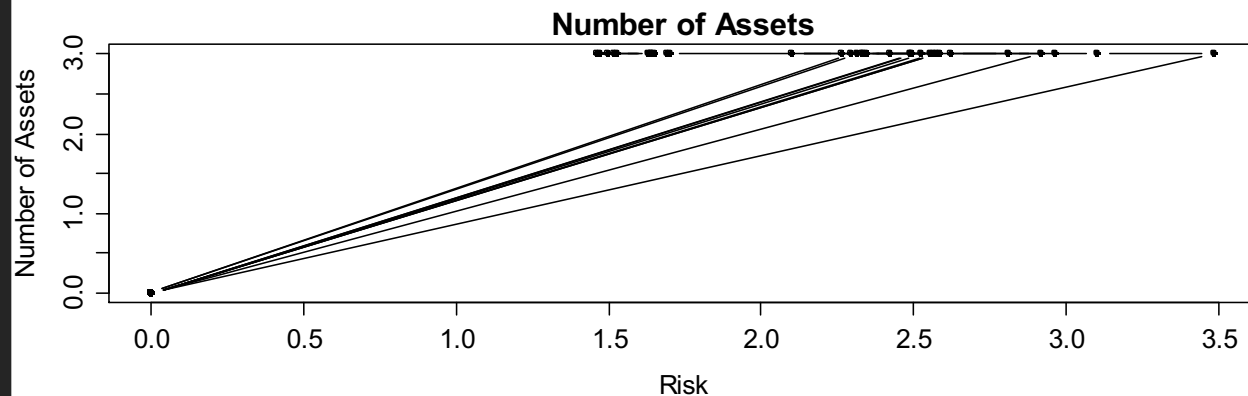
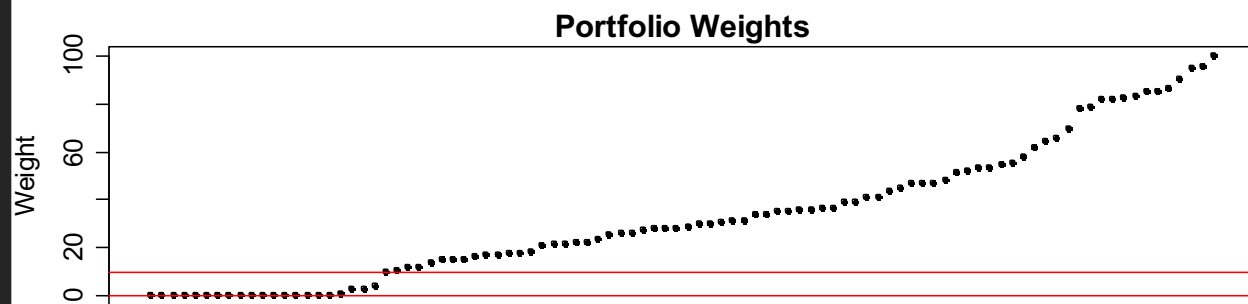


$$\begin{aligned}
 k &= 3 \\
 j &= 7 \\
 \alpha_j &= 0 \\
 \beta_j &= 1
 \end{aligned}$$



# K-means.IND portfolio

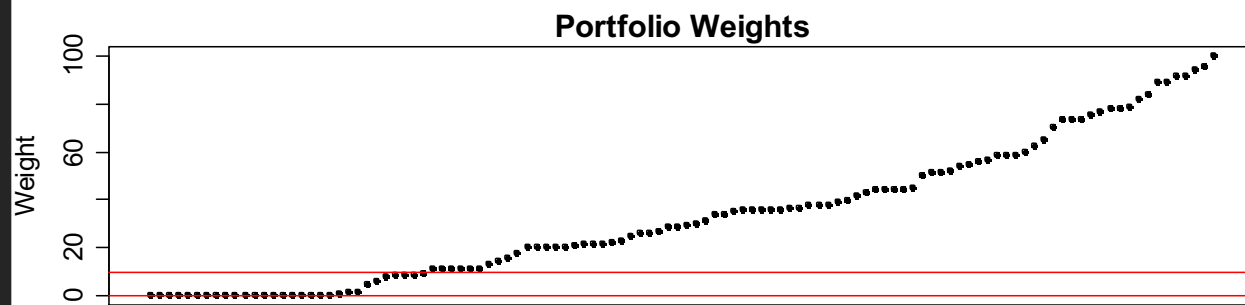
$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j}^n b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$



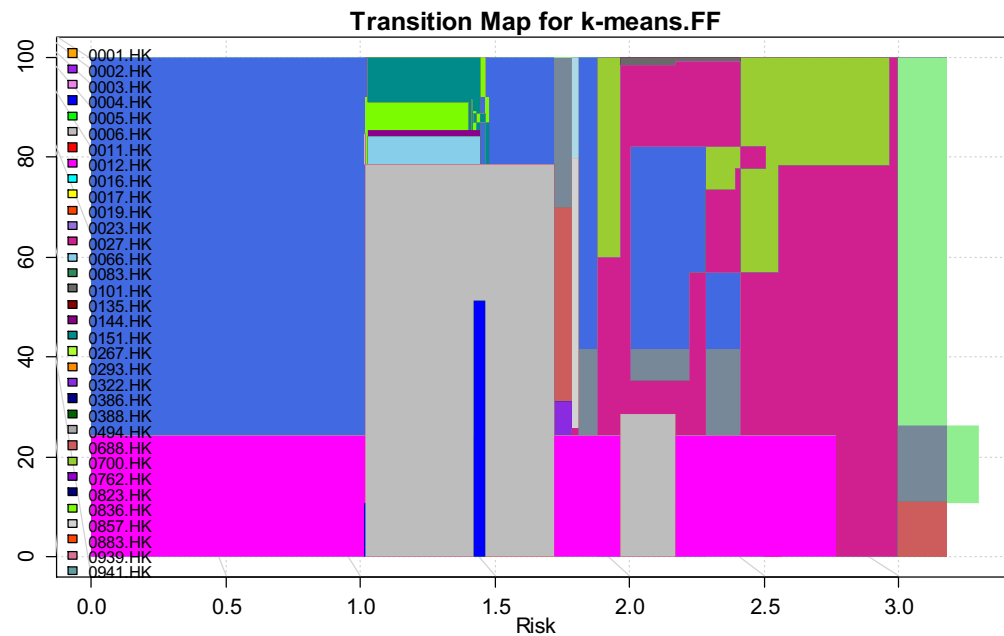
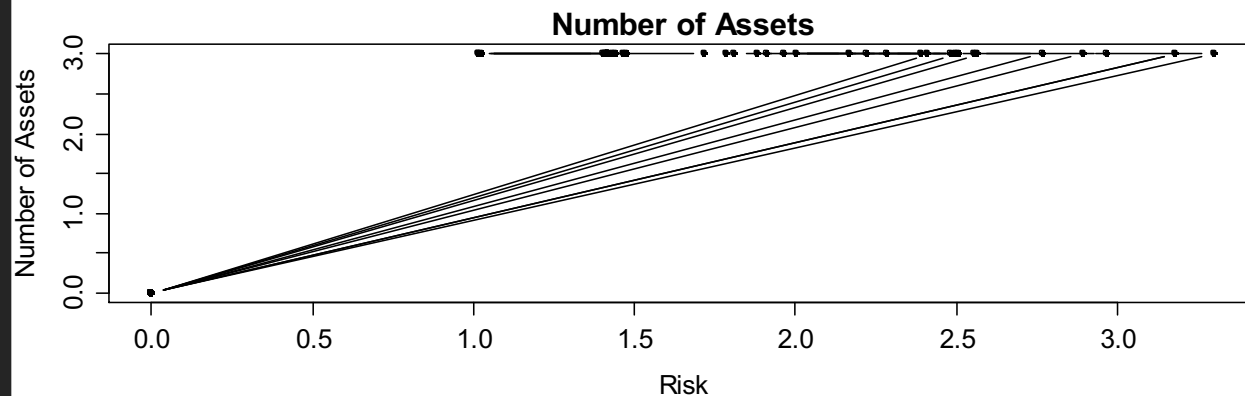
$$\begin{aligned}
 k &= 3 \\
 j &= 7 \\
 \alpha_j &= 0 \\
 \beta_j &= 1
 \end{aligned}$$

# K-means.FF portfolio

$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j}^n b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$



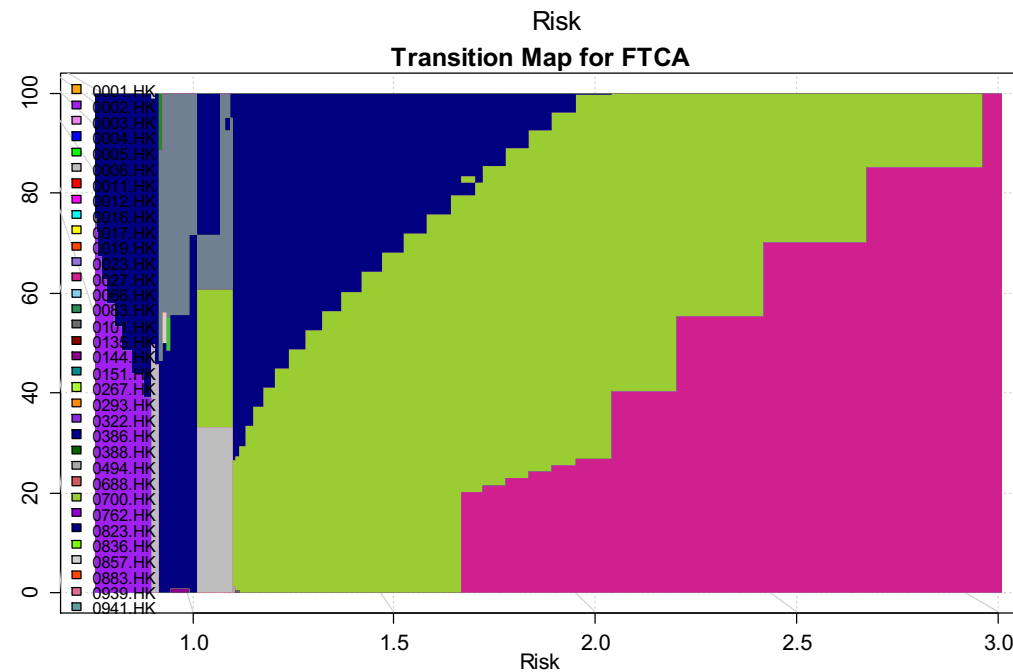
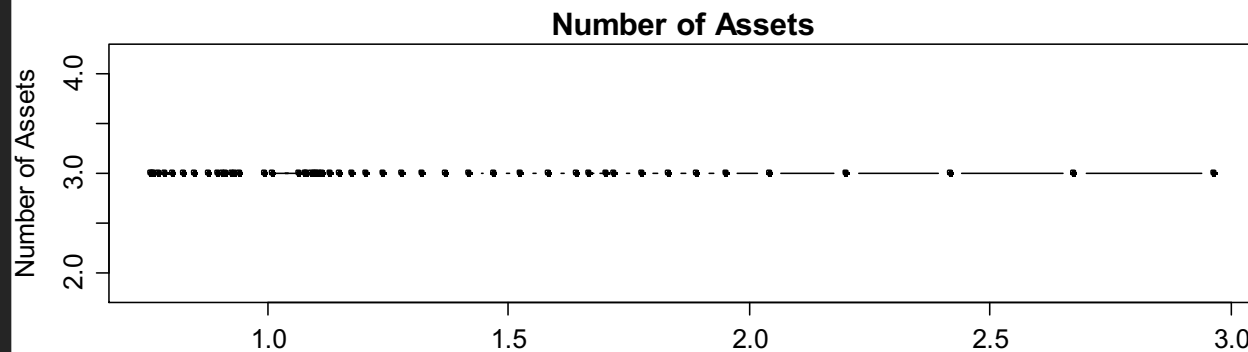
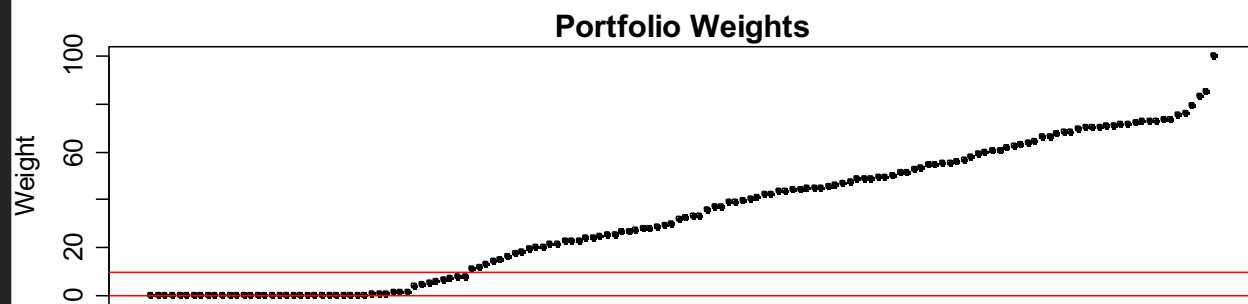
$$\begin{aligned}
 k &= 3 \\
 j &= 7 \\
 \alpha_j &= 0 \\
 \beta_j &= 1
 \end{aligned}$$





# K-means.FTCA portfolio

$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j} b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$



$$\begin{aligned}
 k &= 3 \\
 j &= 18 \\
 \alpha_j &= 0 \\
 \beta_j &= 1
 \end{aligned}$$

# CCMV portfolio

$$\min_x x' Q x$$

$$\text{s.t. } r'x \geq \bar{r}$$

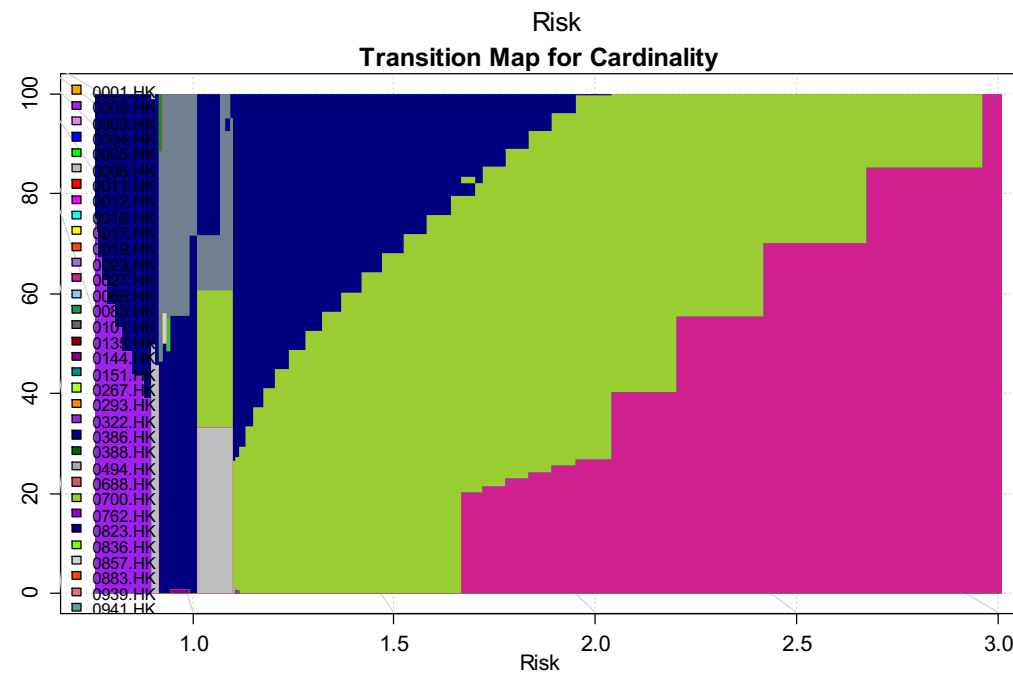
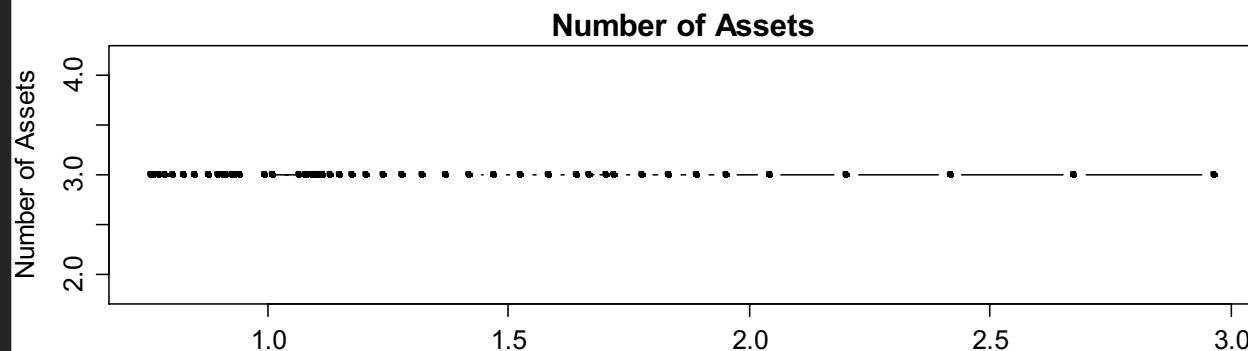
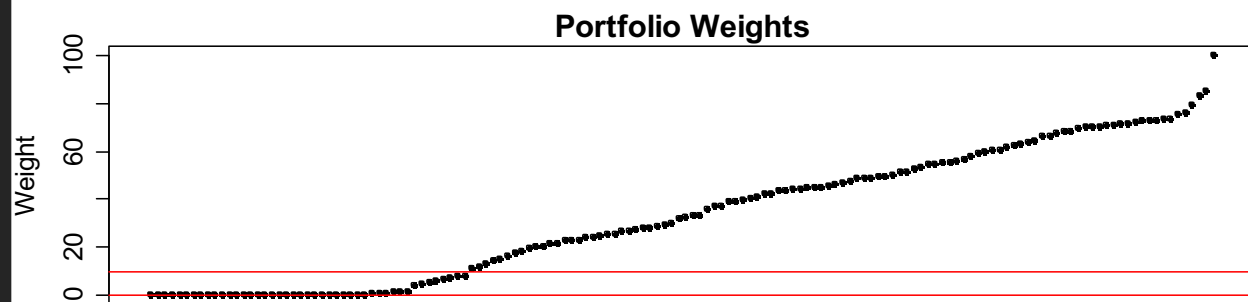
$$\mathbf{1}'x = 1$$

$$0 \leq x_i \leq 1$$

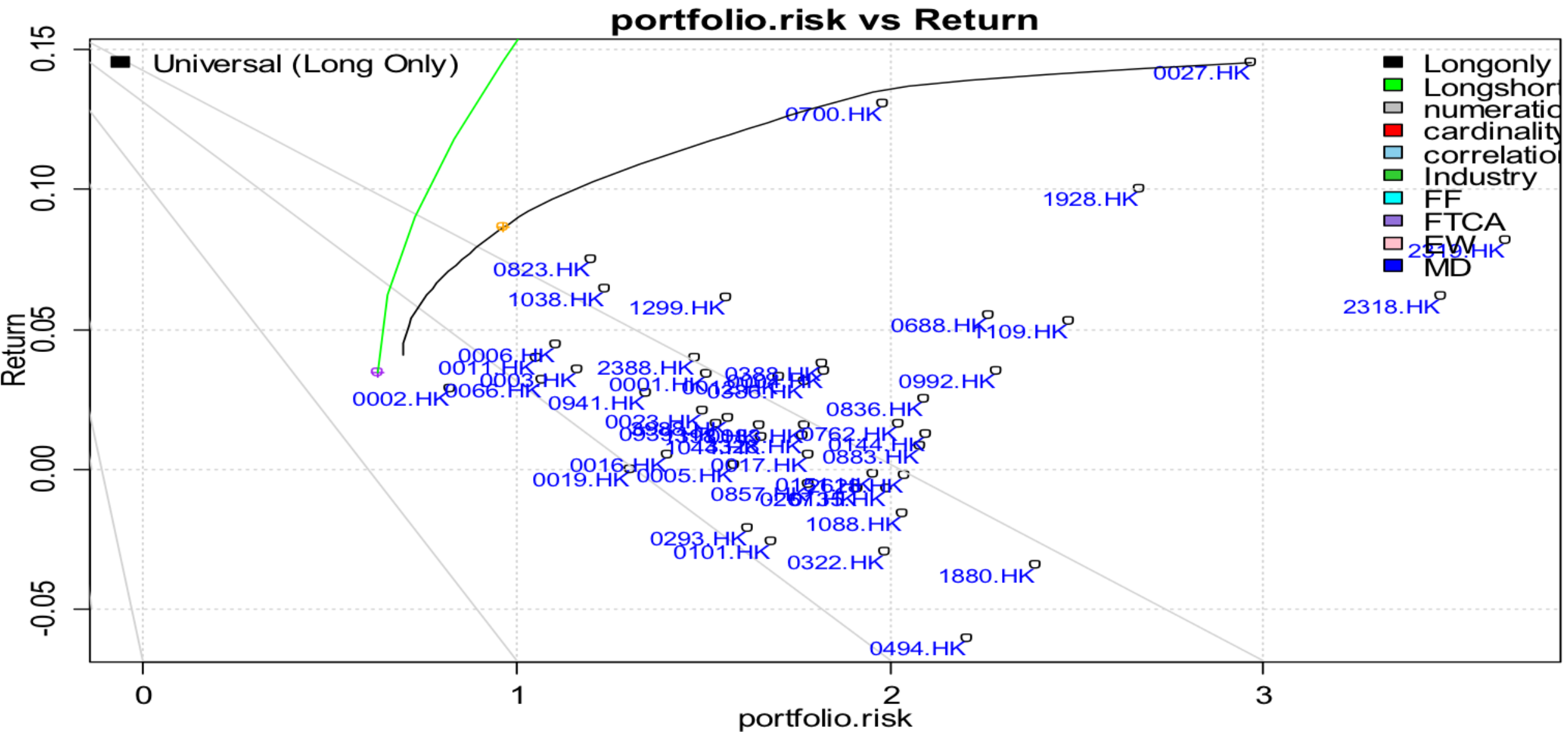
$$\sum_{i=1}^n b_i = k$$

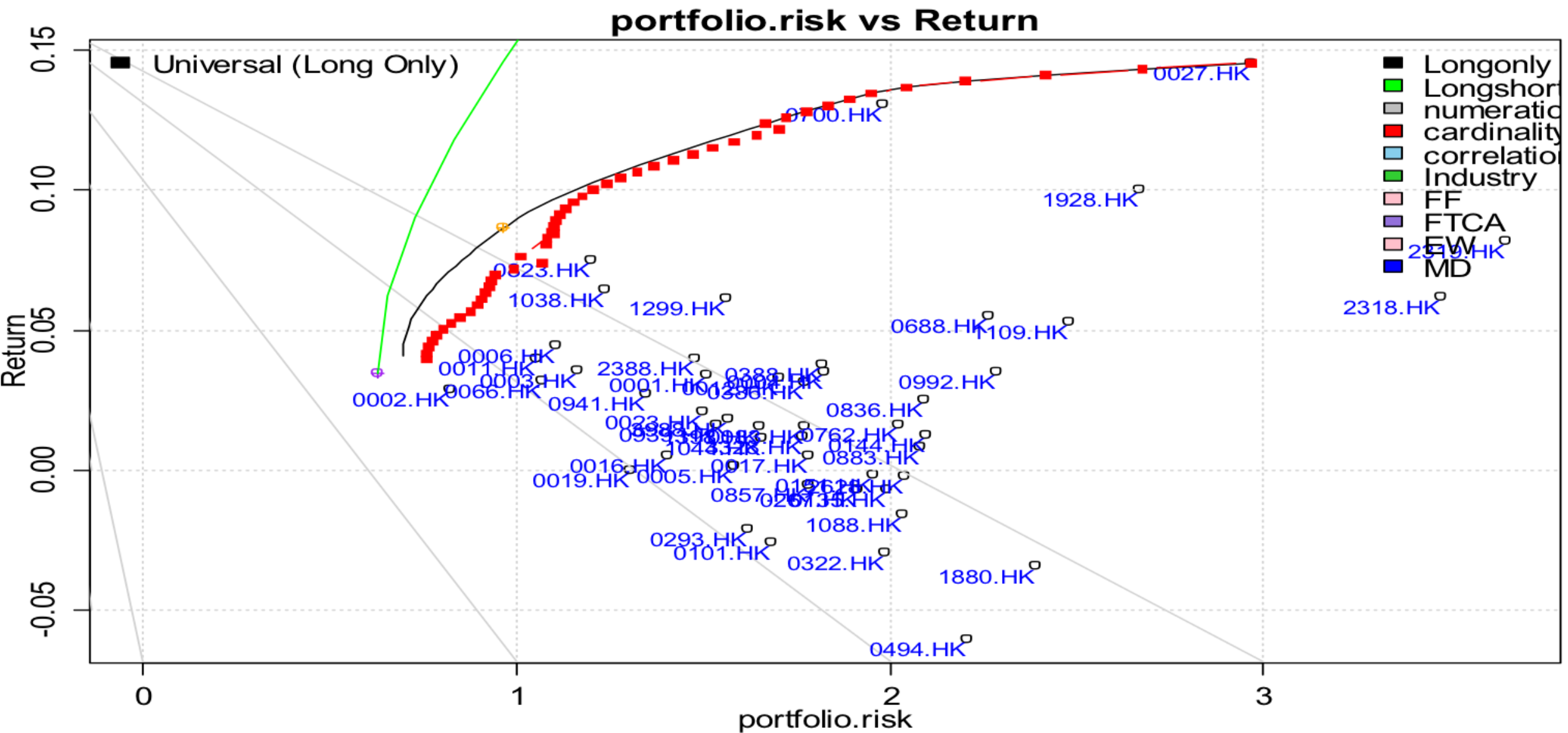
$$b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases}$$

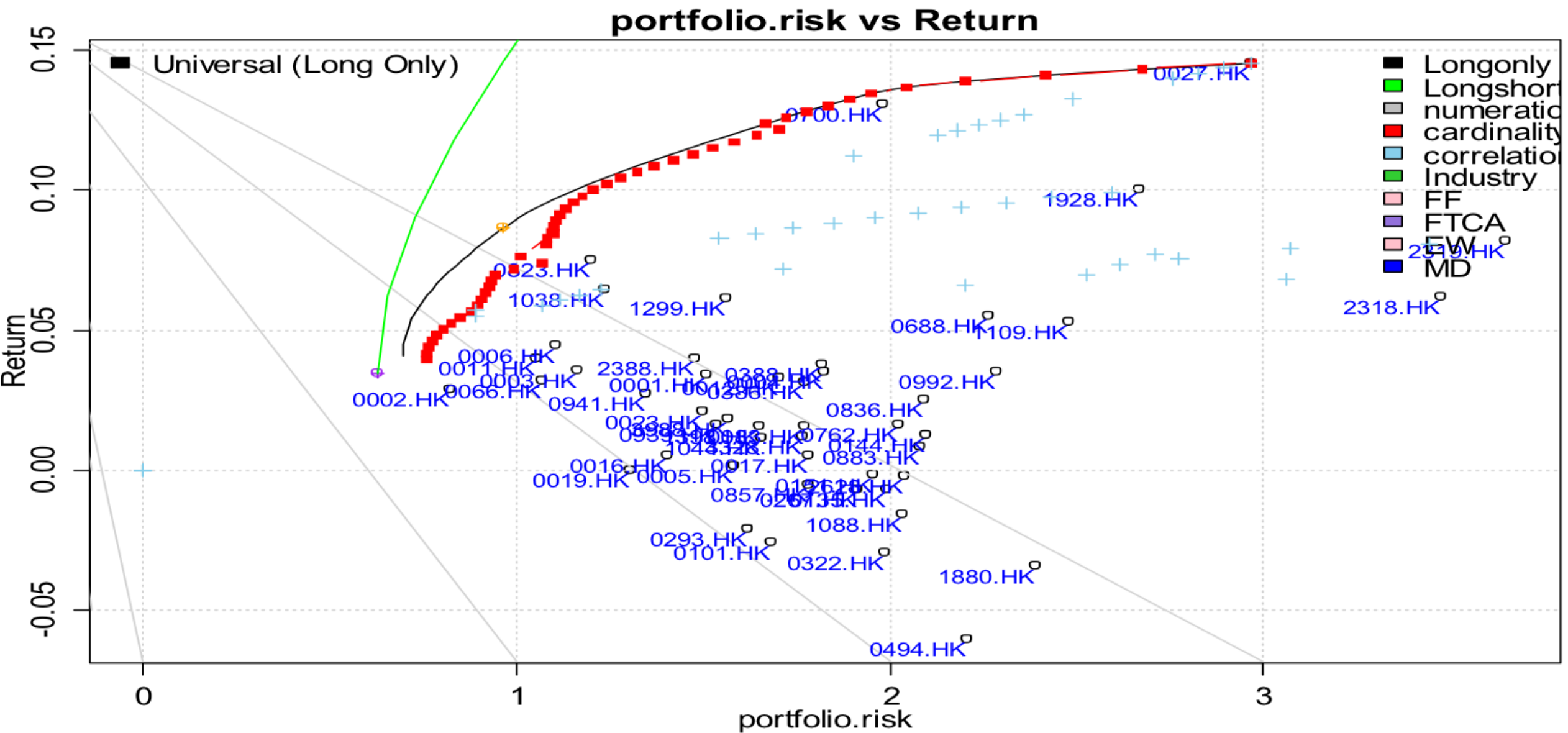
for  $i$  from 1 to  $n$

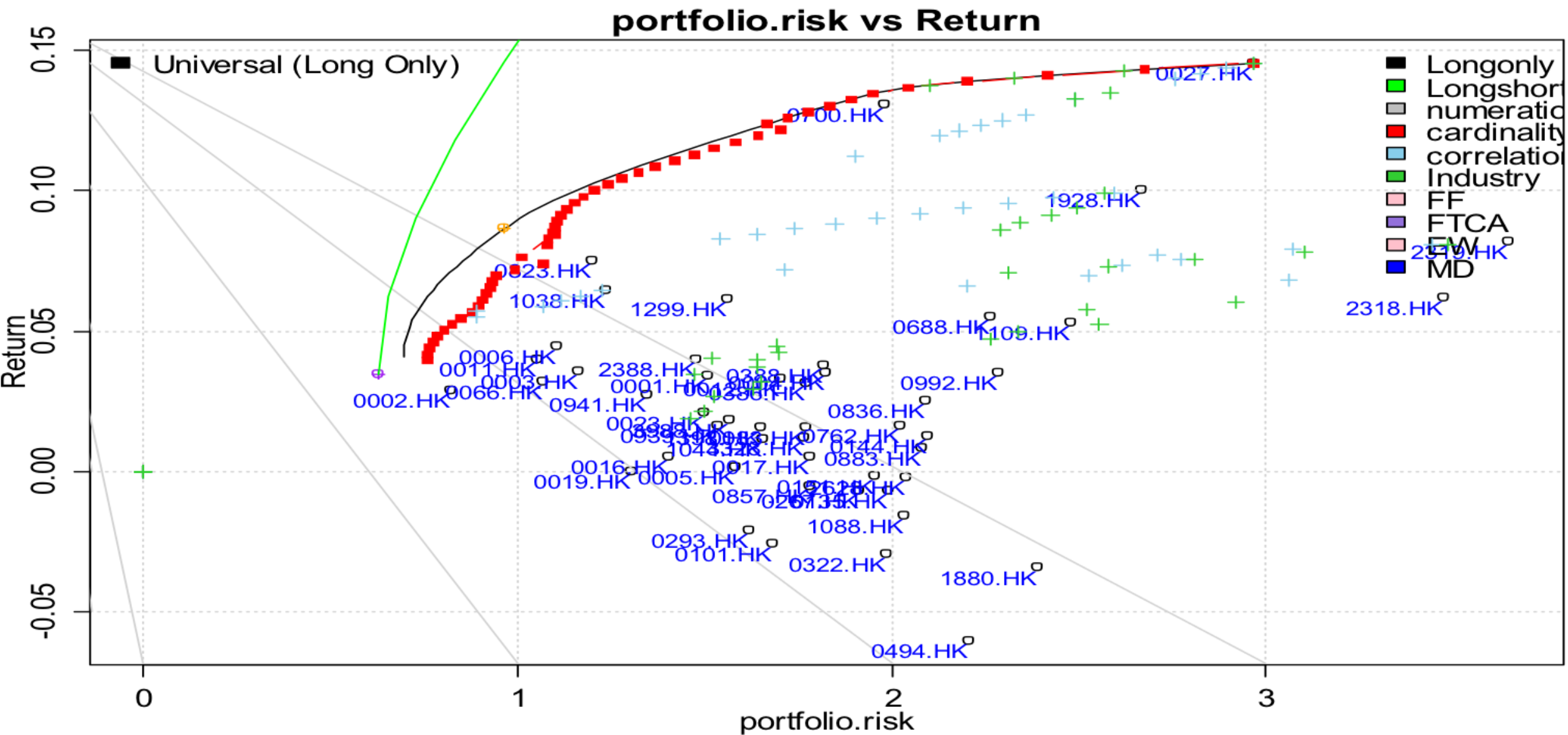


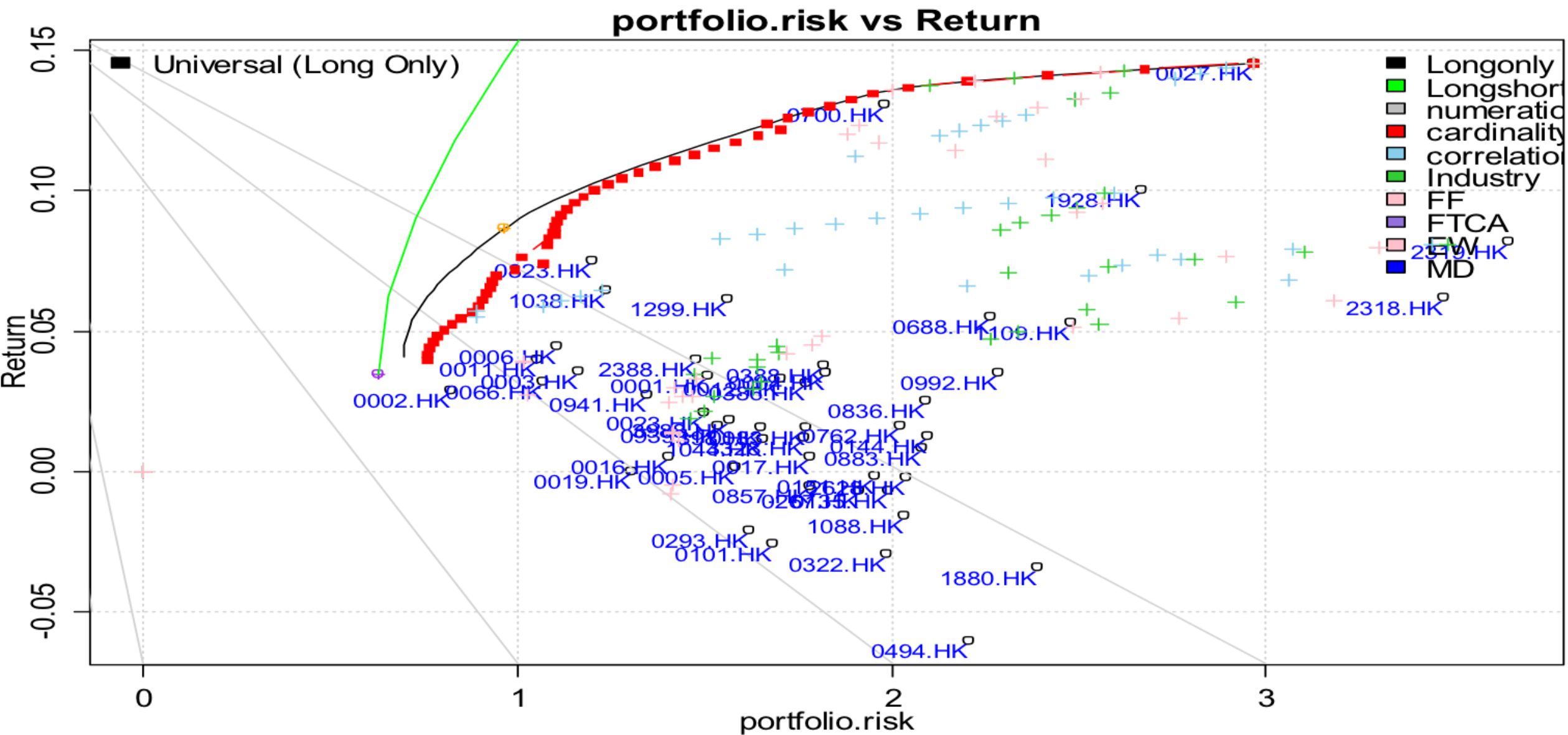
$k = 3$



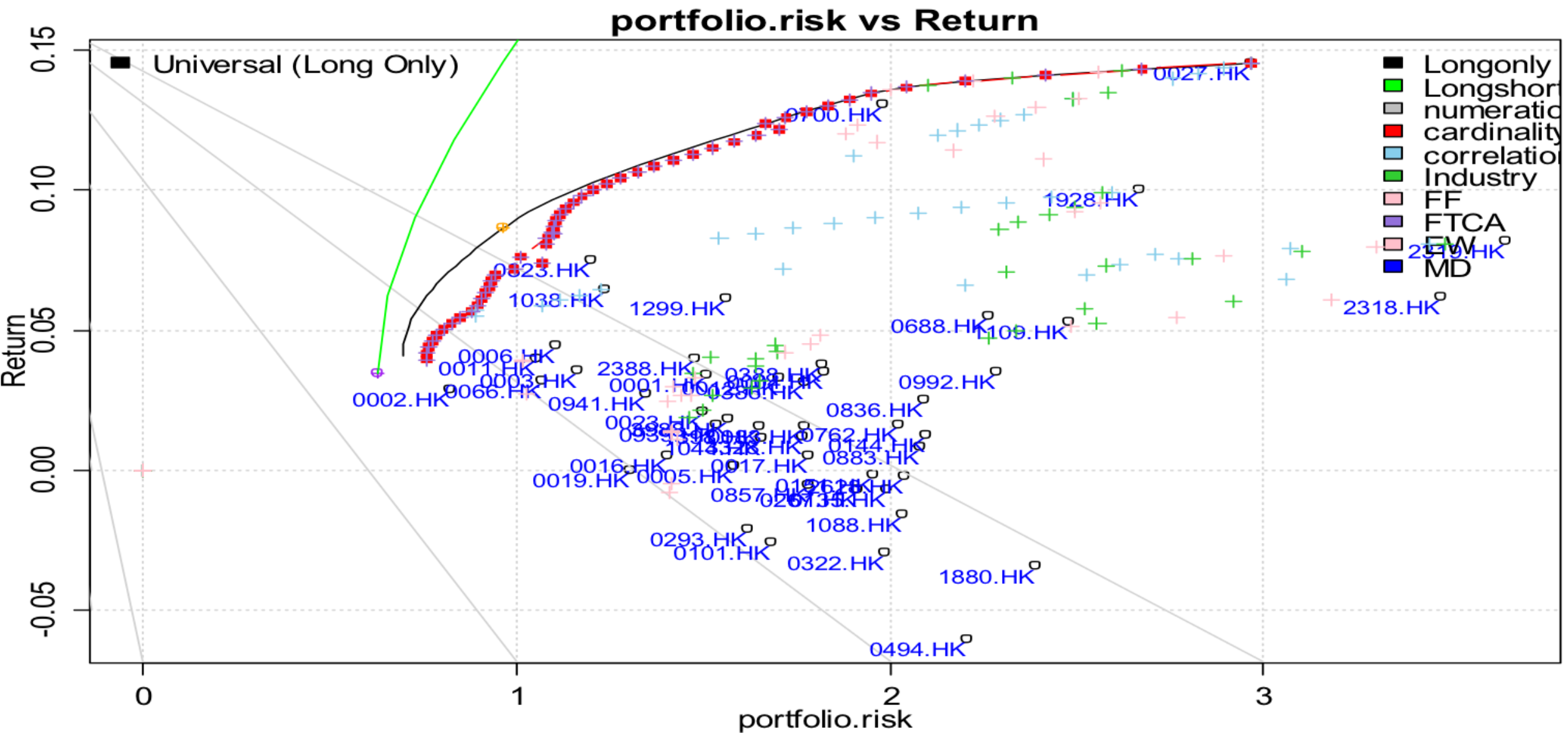






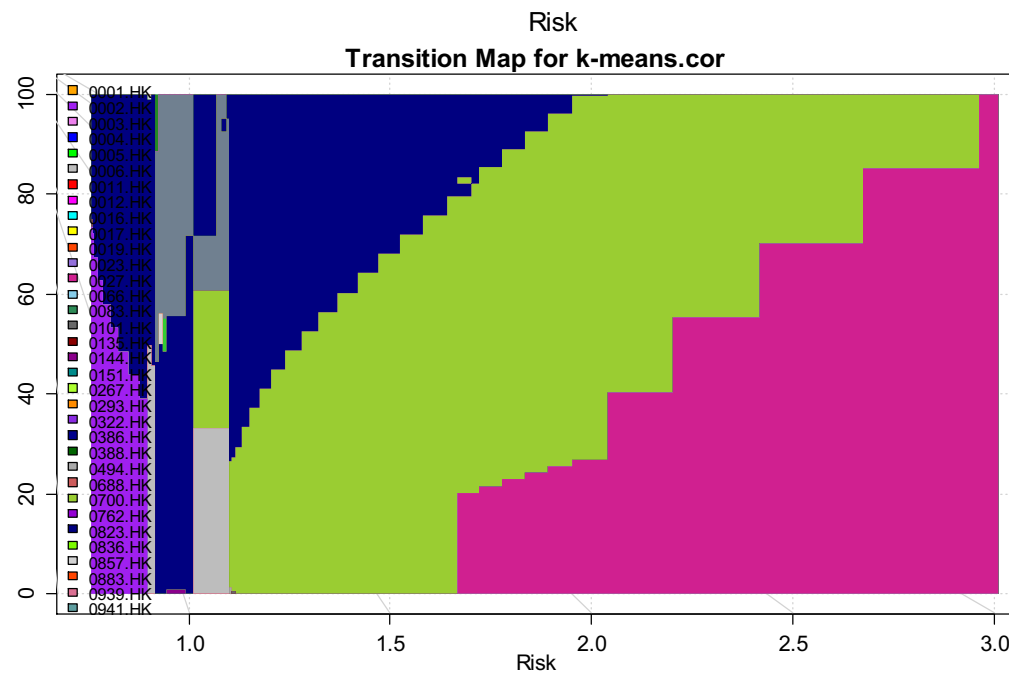
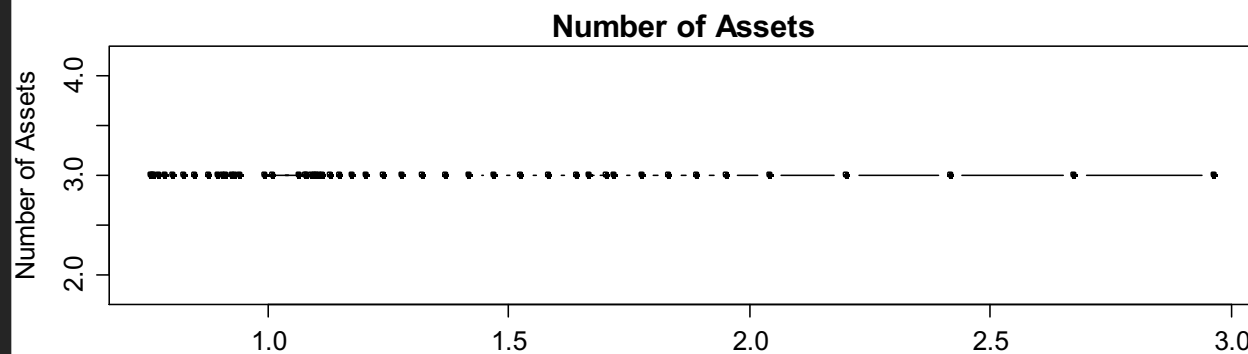
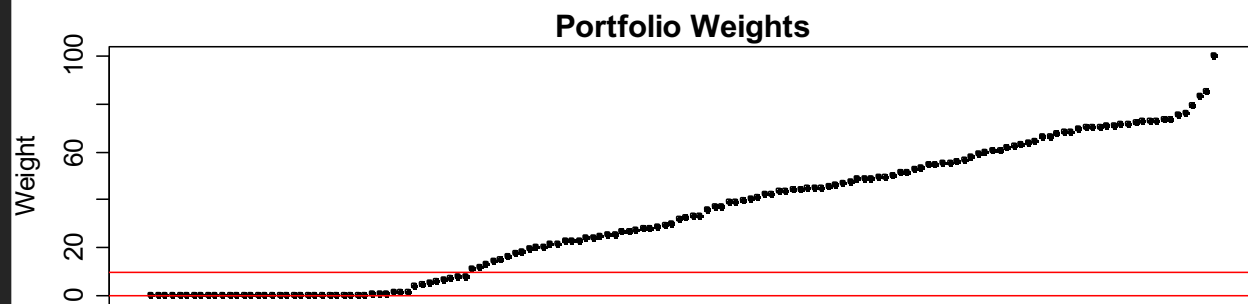






# K-means.cor portfolio

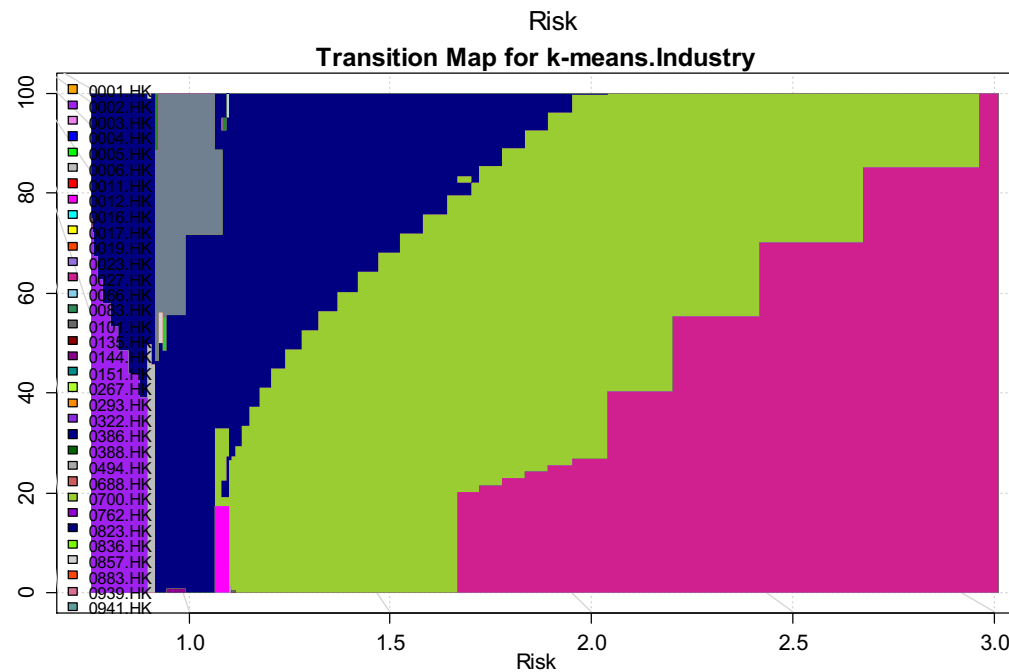
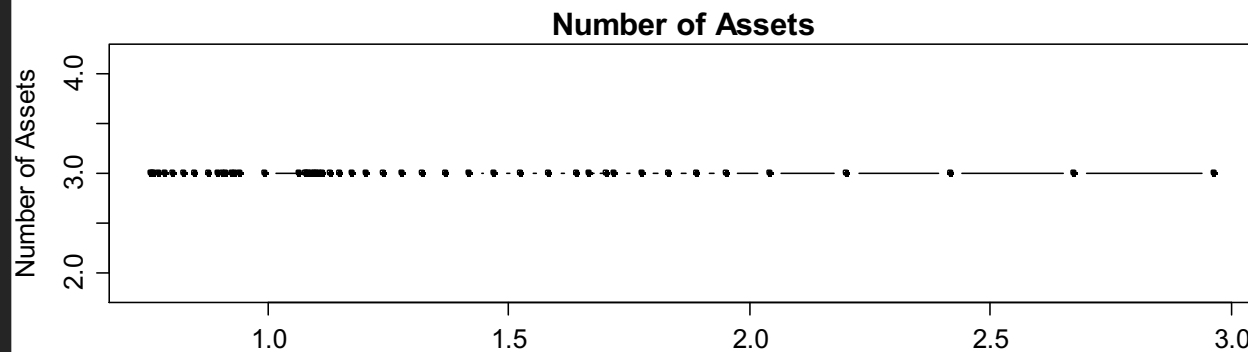
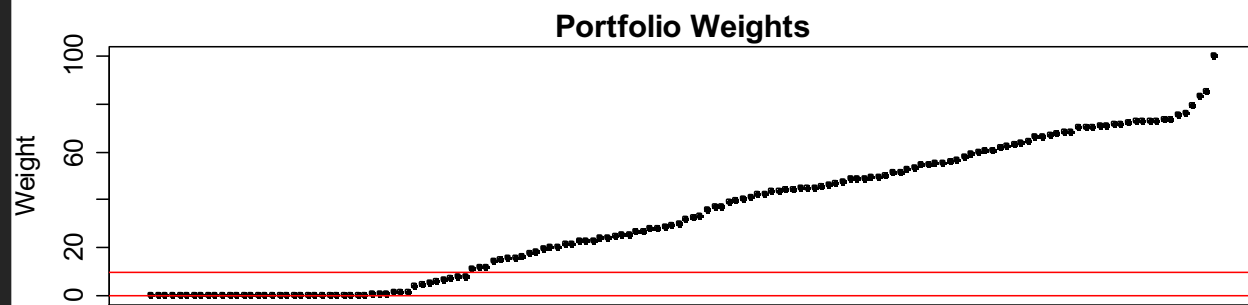
$$\begin{aligned} \min_x & x' Q x \\ \text{s.t. } & r' x \geq \bar{r} \\ & 1' x = 1 \\ & \alpha_j \leq \sum_{i \in I_j} b_i \leq \beta_j \\ & \sum_{i=1}^n b_i = k \\ b_i = & \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\ & i = 1, 2, \dots, n. \\ & j = 1, 2, \dots, k. \end{aligned}$$



$k = 3$   
 $j = 7$   
 $\alpha_j = 0$   
 $\beta_j = 3$

# K-means.IND portfolio

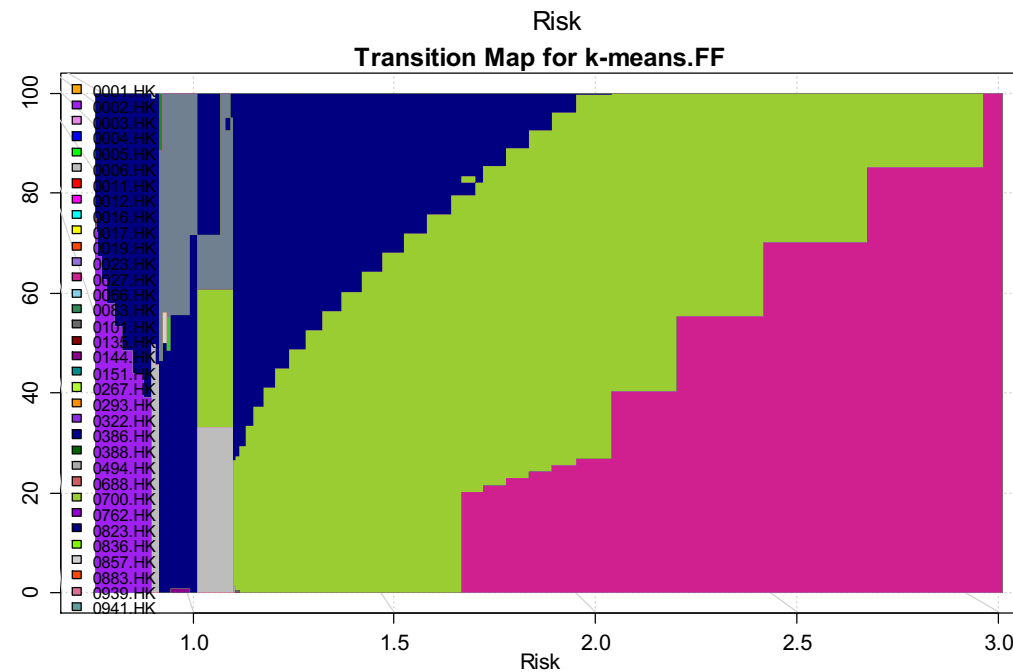
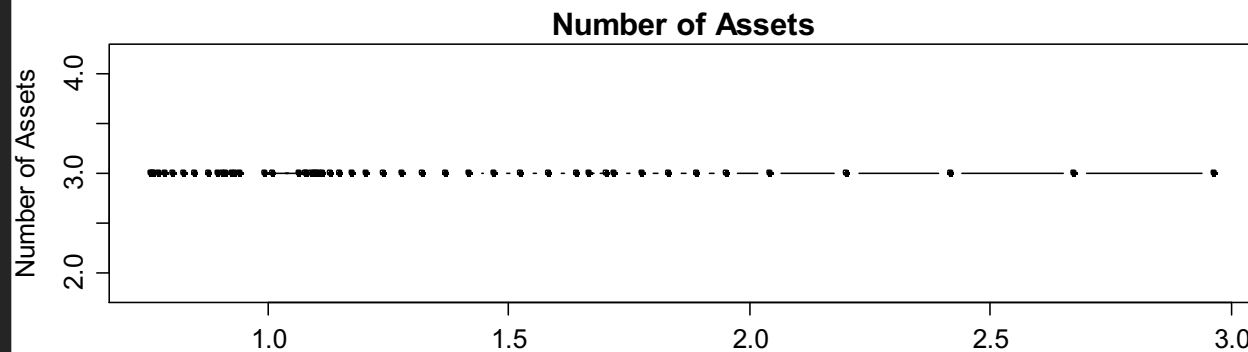
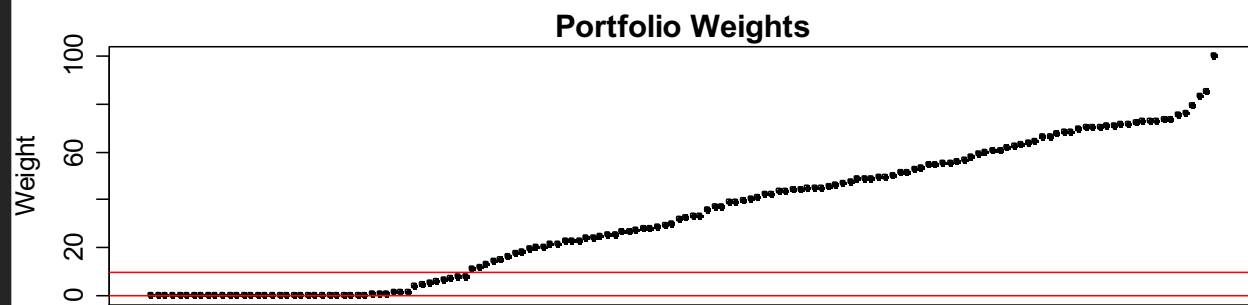
$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j} b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$



$$\begin{aligned}
 k &= 3 \\
 j &= 7 \\
 \alpha_j &= 0 \\
 \beta_j &= 3
 \end{aligned}$$

# K-means.FF portfolio

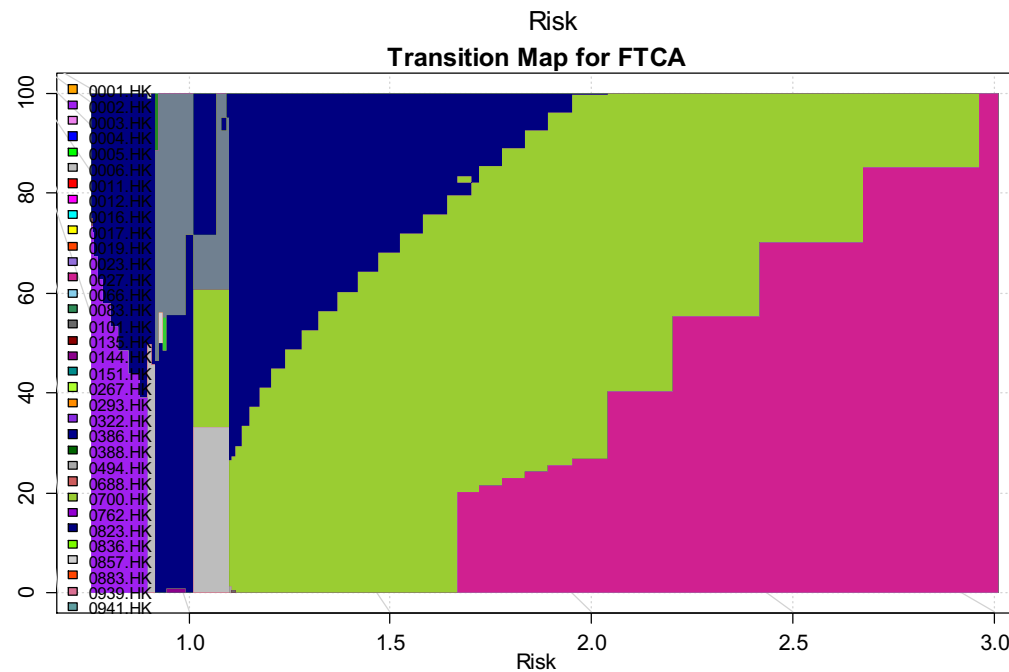
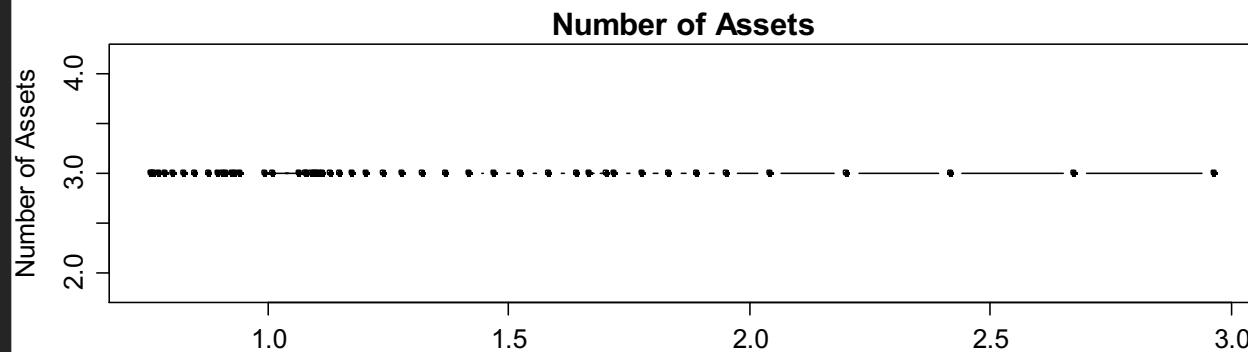
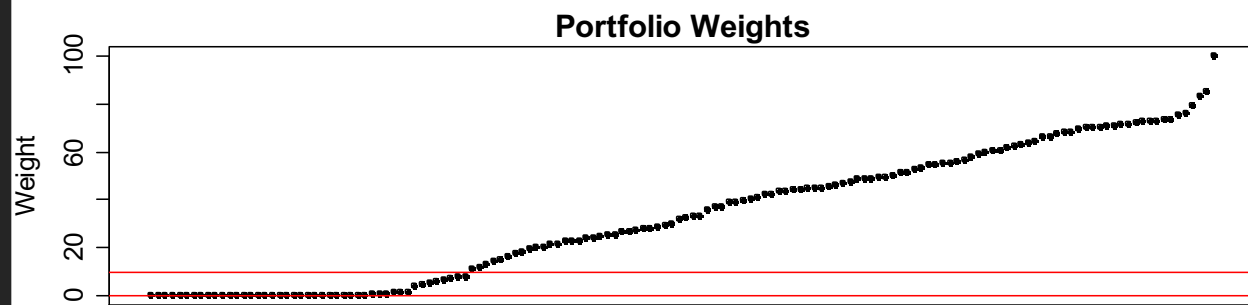
$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j} b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$



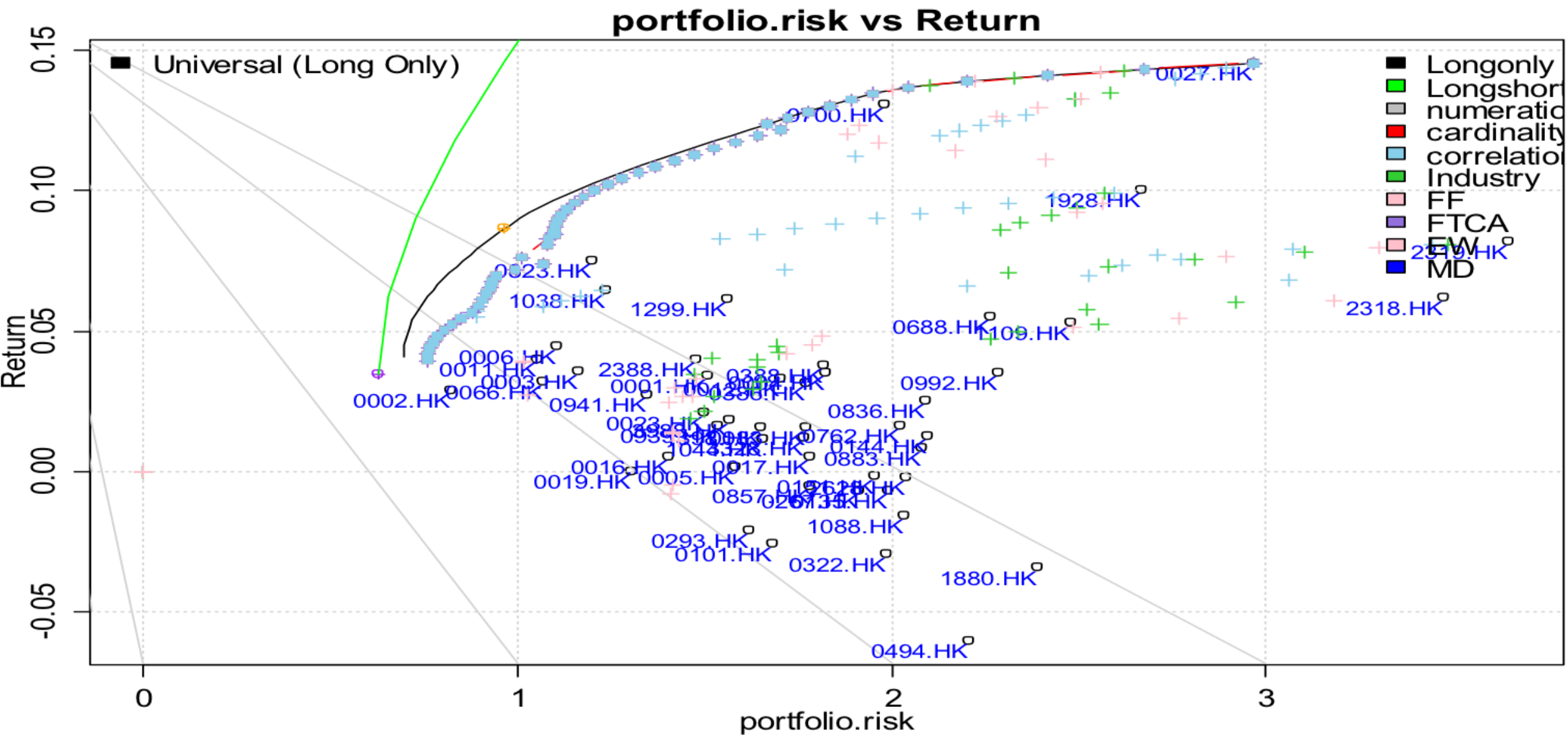
$$\begin{aligned}
 k &= 3 \\
 j &= 7 \\
 \alpha_j &= 0 \\
 \beta_j &= 3
 \end{aligned}$$

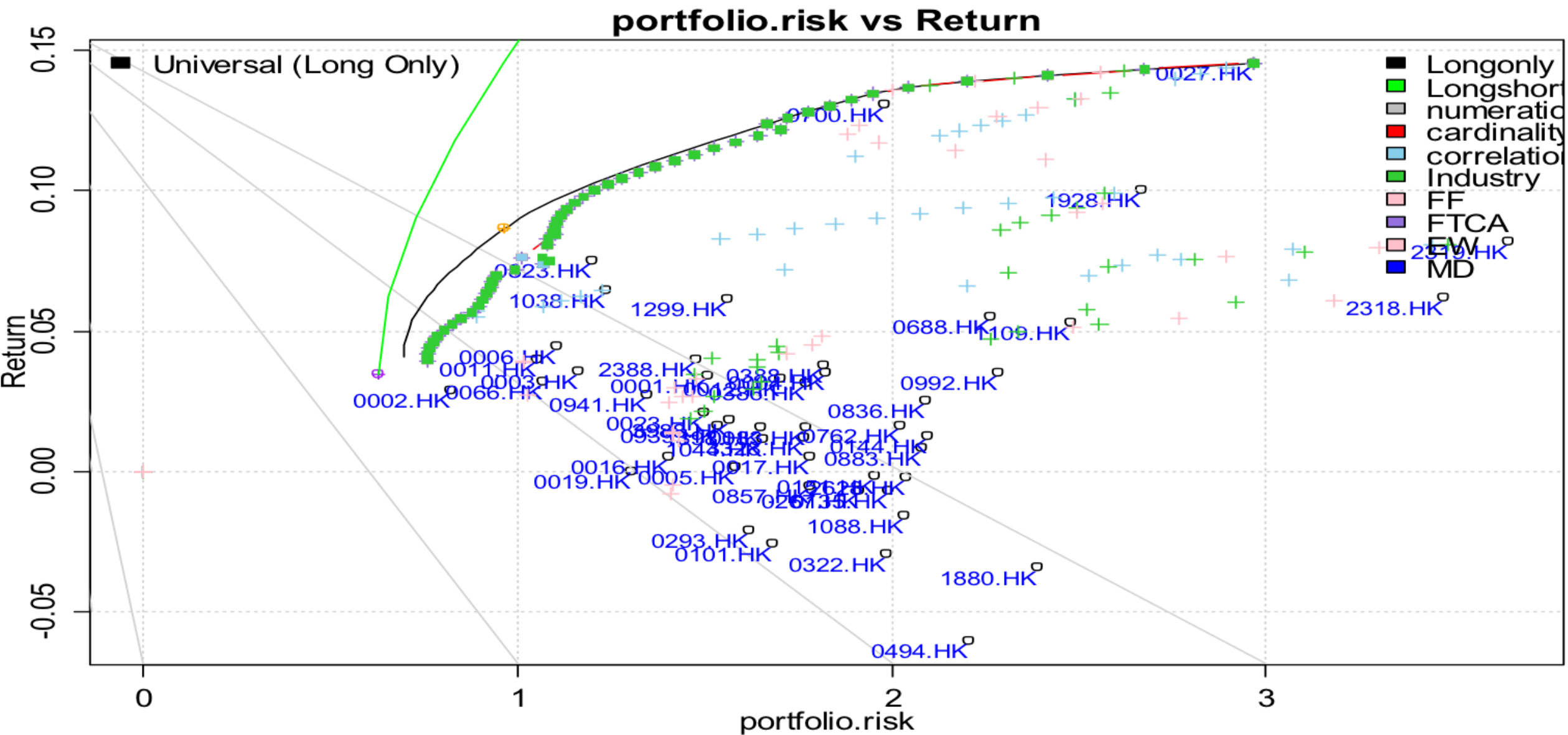
# K-means.FTCA portfolio

$$\begin{aligned}
 & \min_x x' Q x \\
 & \text{s.t. } r'x \geq \bar{r} \\
 & \mathbf{1}'x = 1 \\
 & \alpha_j \leq \sum_{i \in I_j}^n b_i \leq \beta_j \\
 & \sum_{i=1}^n b_i = k \\
 & b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases} \\
 & i = 1, 2, \dots, n. \\
 & j = 1, 2, \dots, k.
 \end{aligned}$$

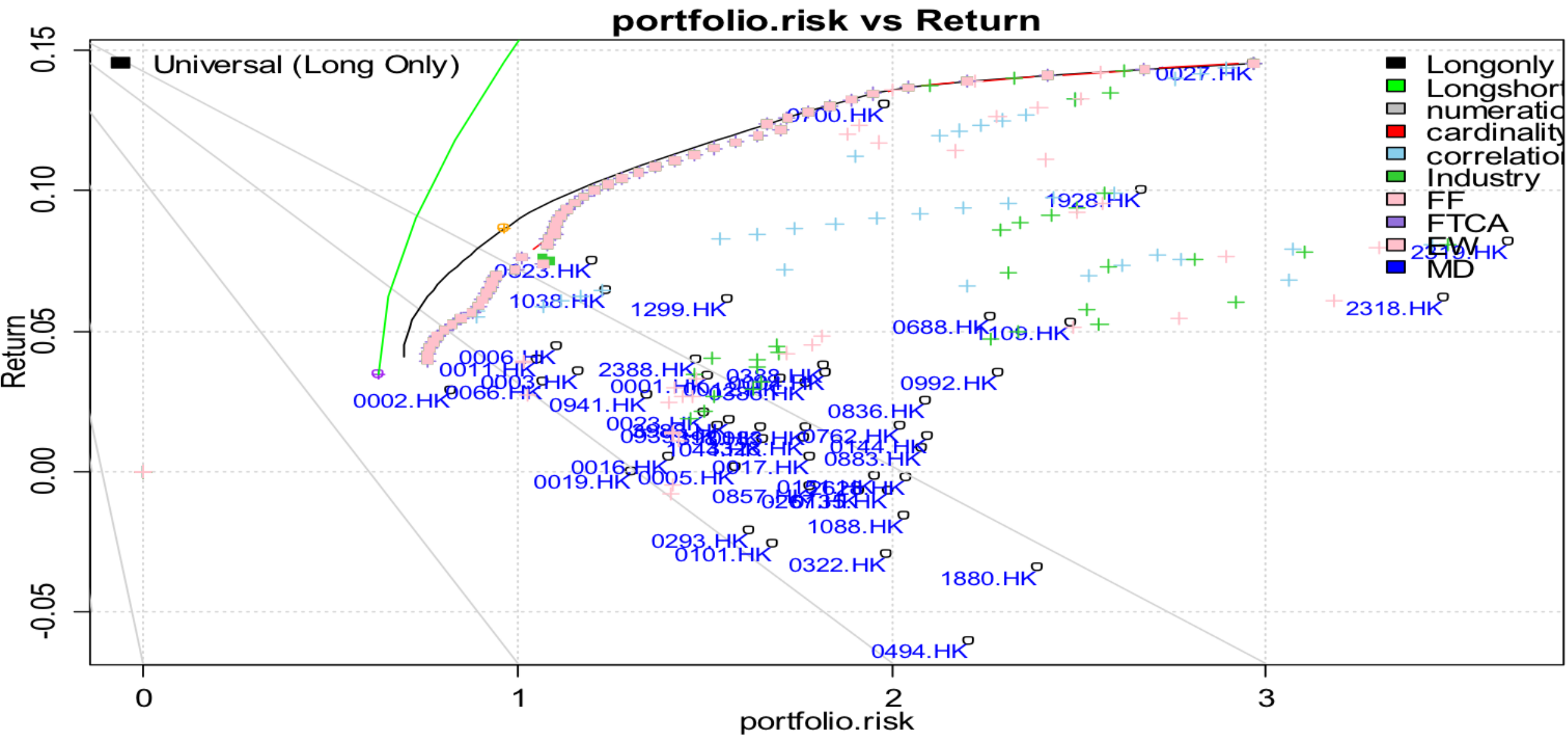


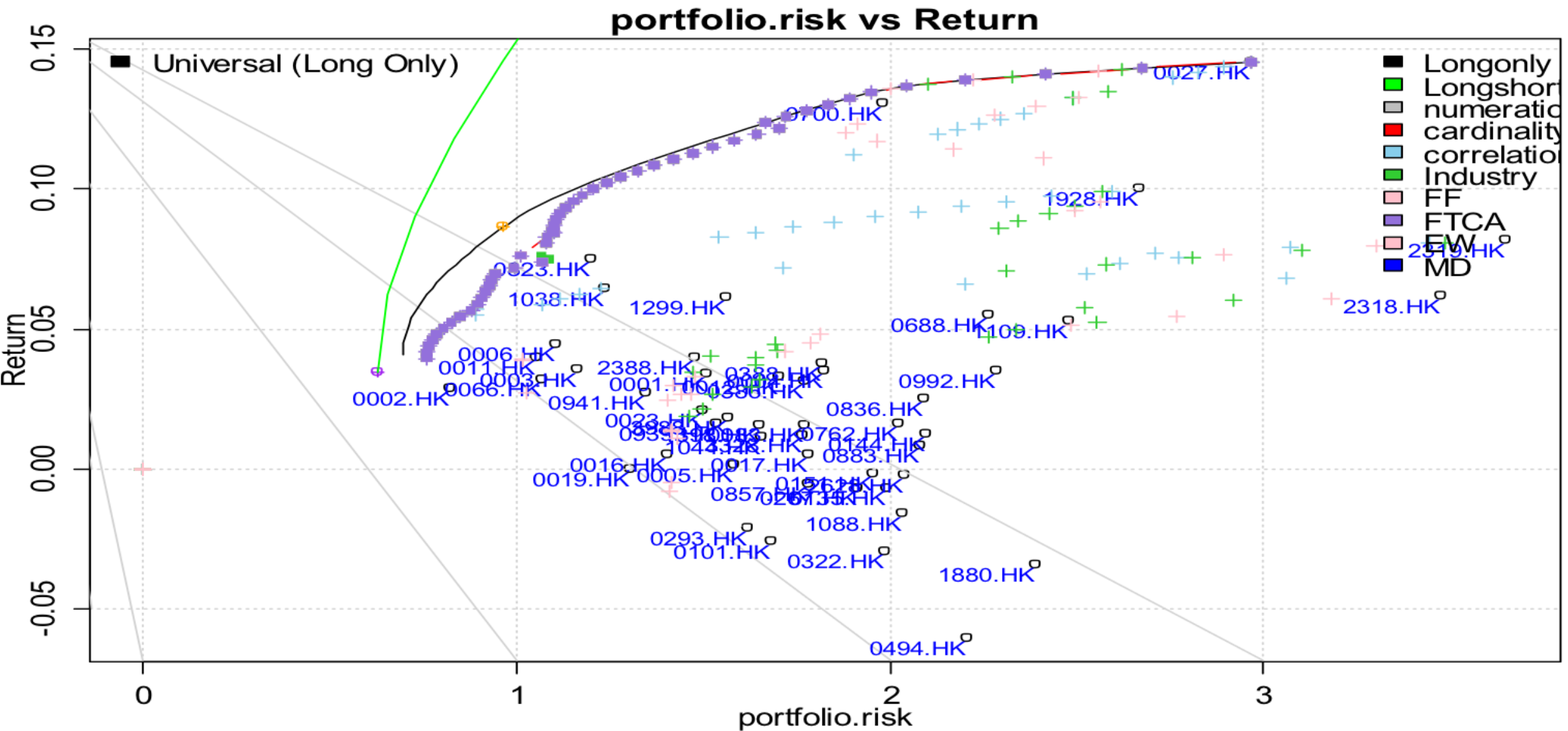
$$\begin{aligned}
 k &= 3 \\
 j &= 18 \\
 \alpha_j &= 0 \\
 \beta_j &= 3
 \end{aligned}$$

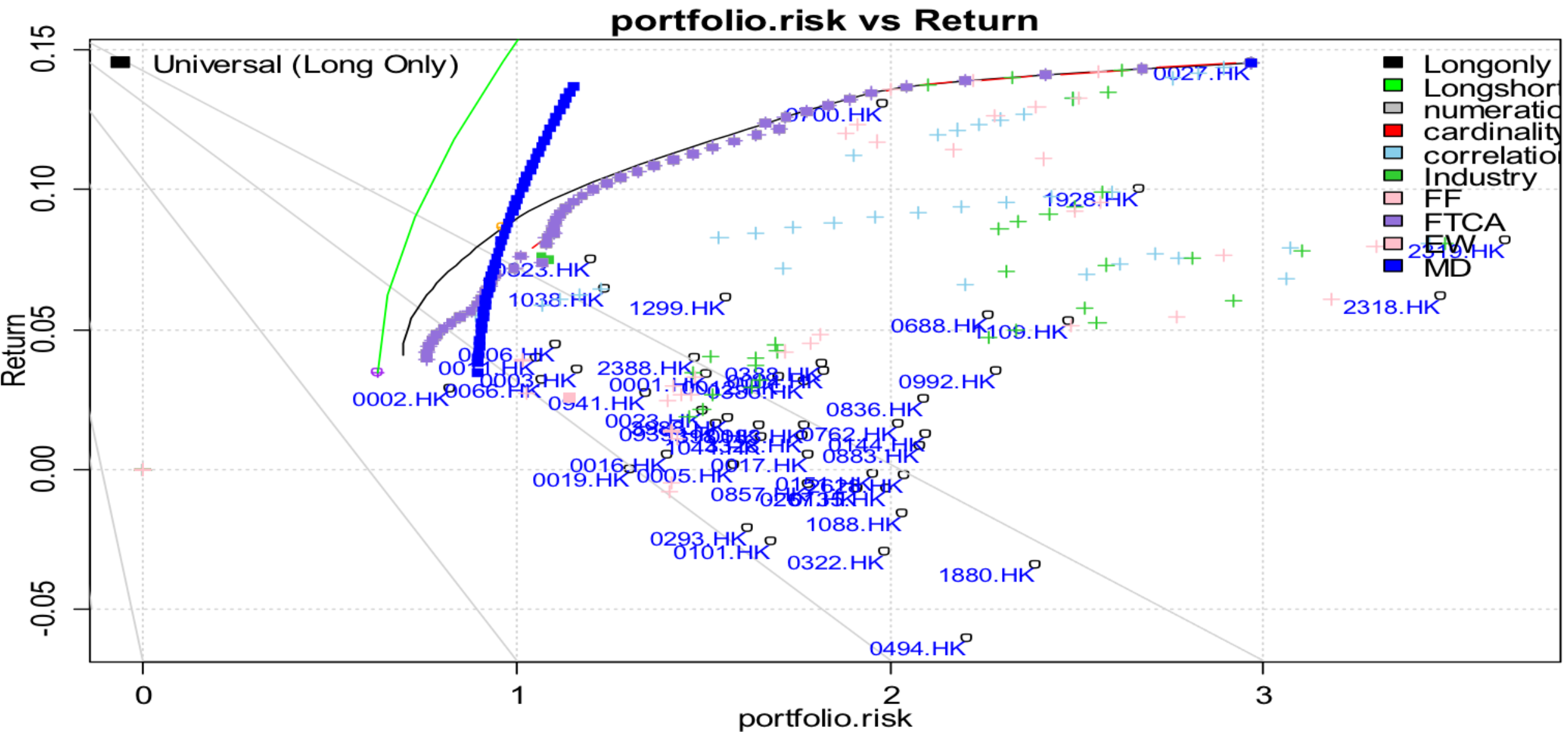














# Static?

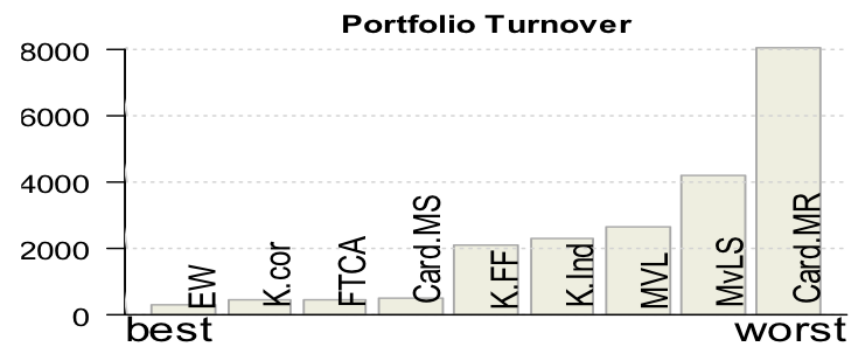
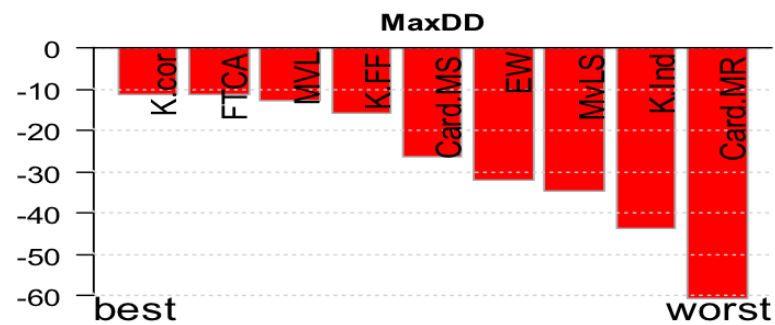
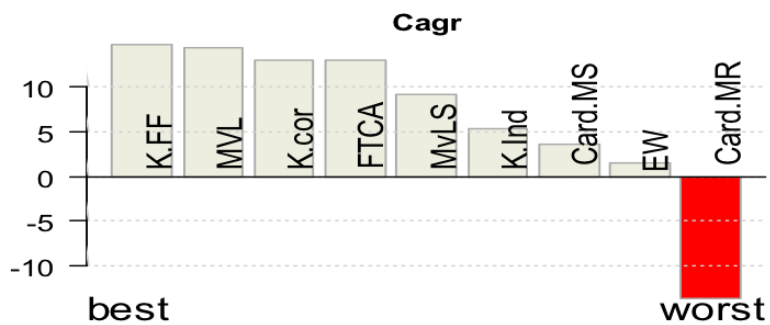
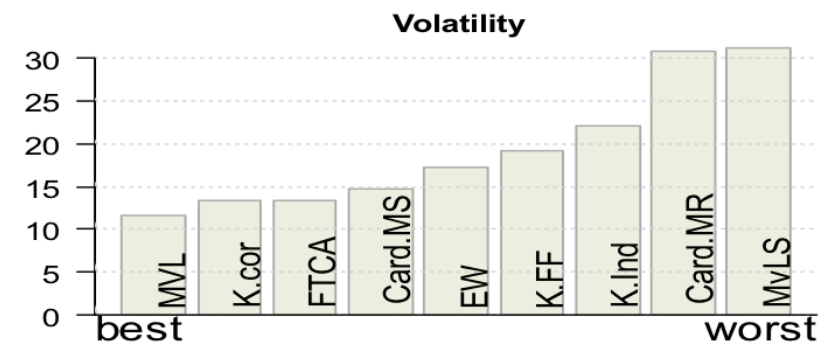
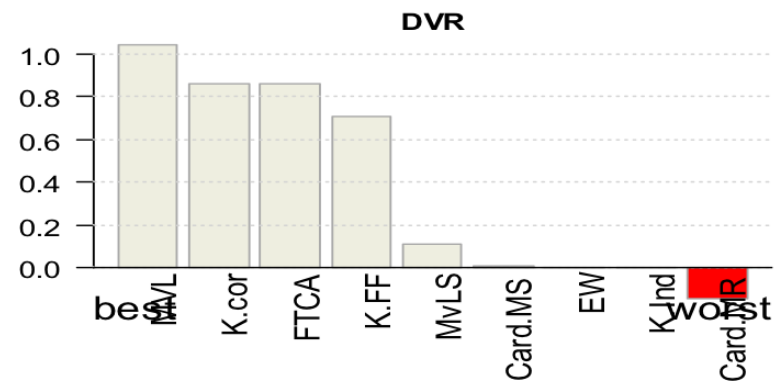
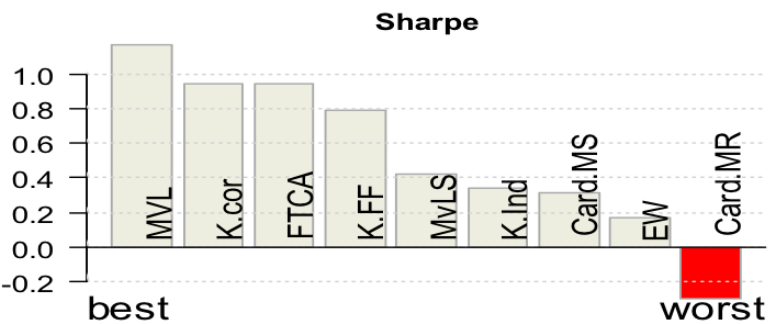
Let history have a say!



# Back testing

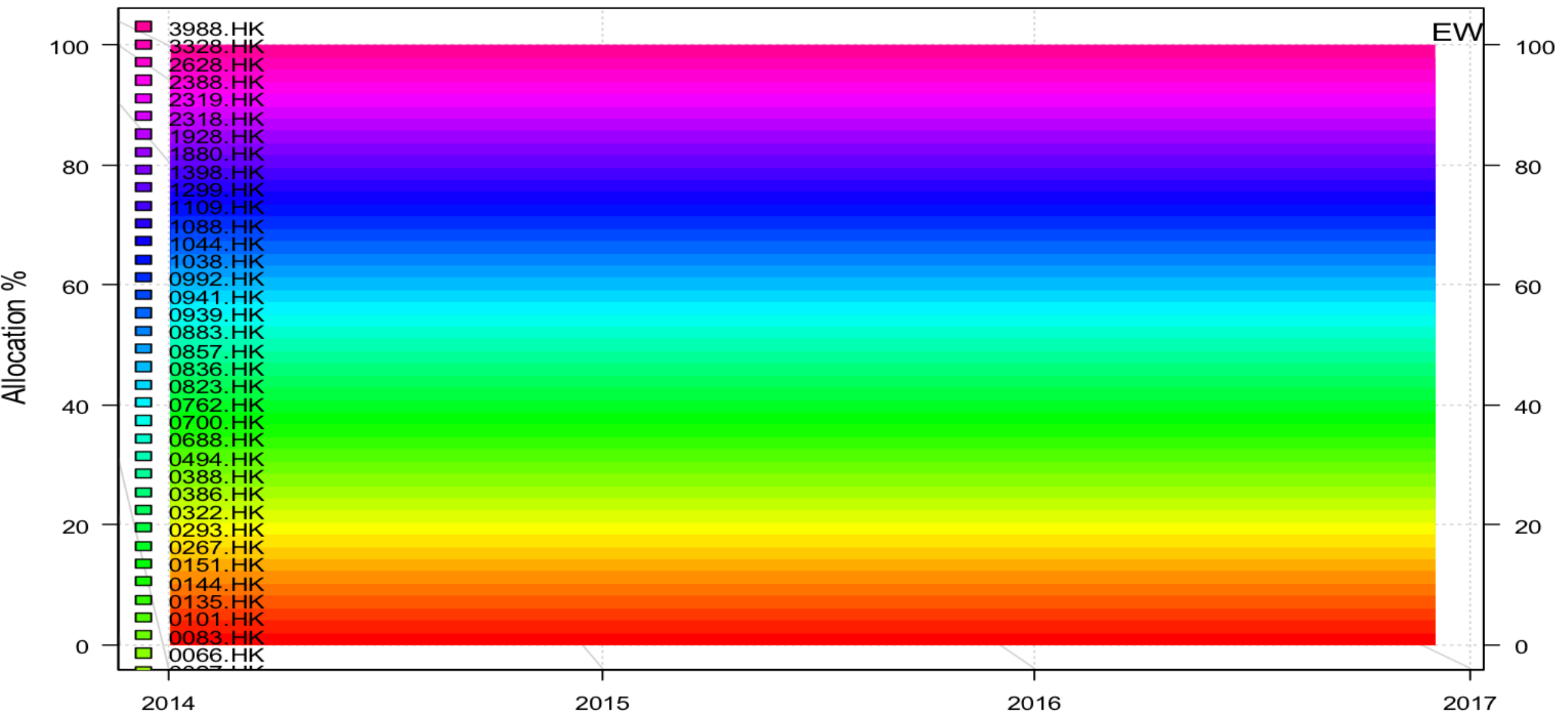
Portfolio performance across time







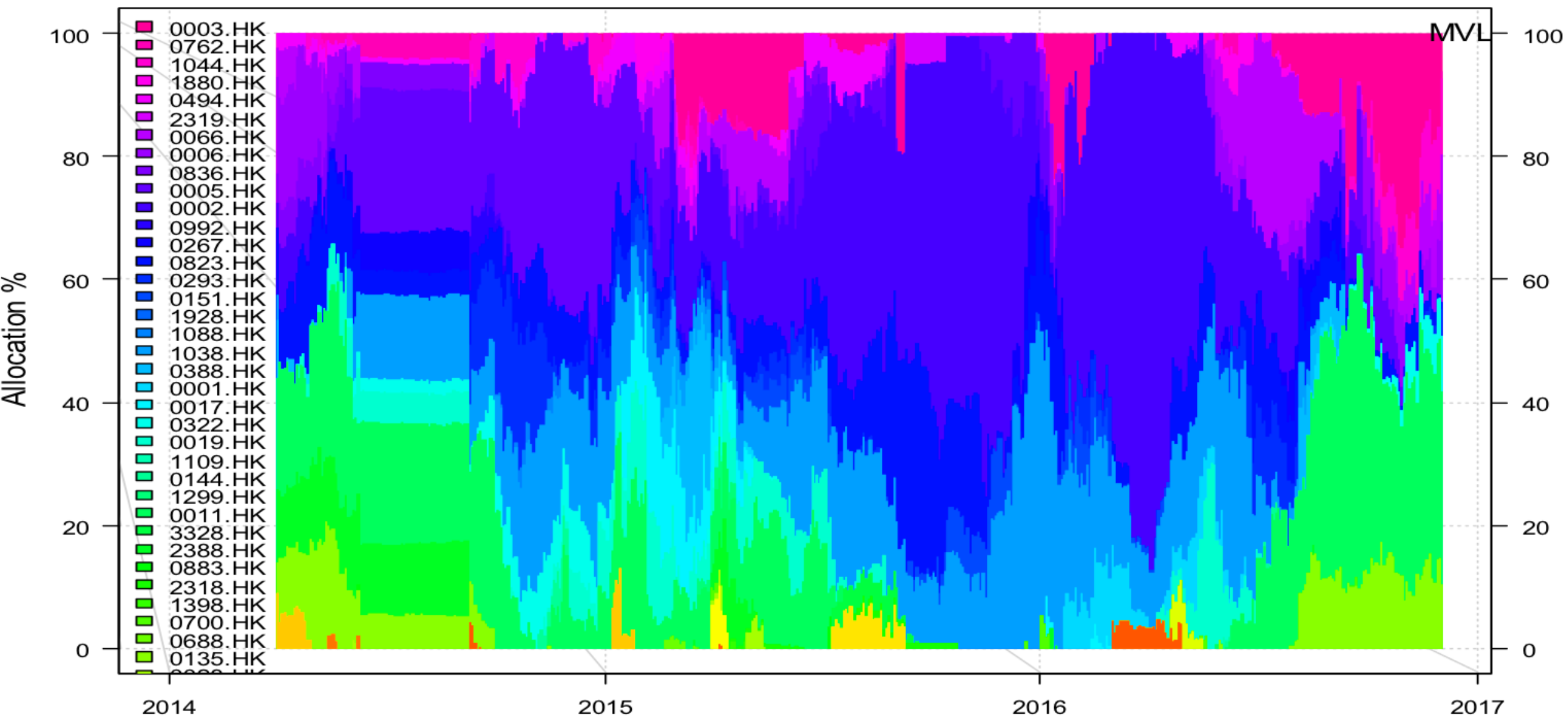
Transition Map for EW



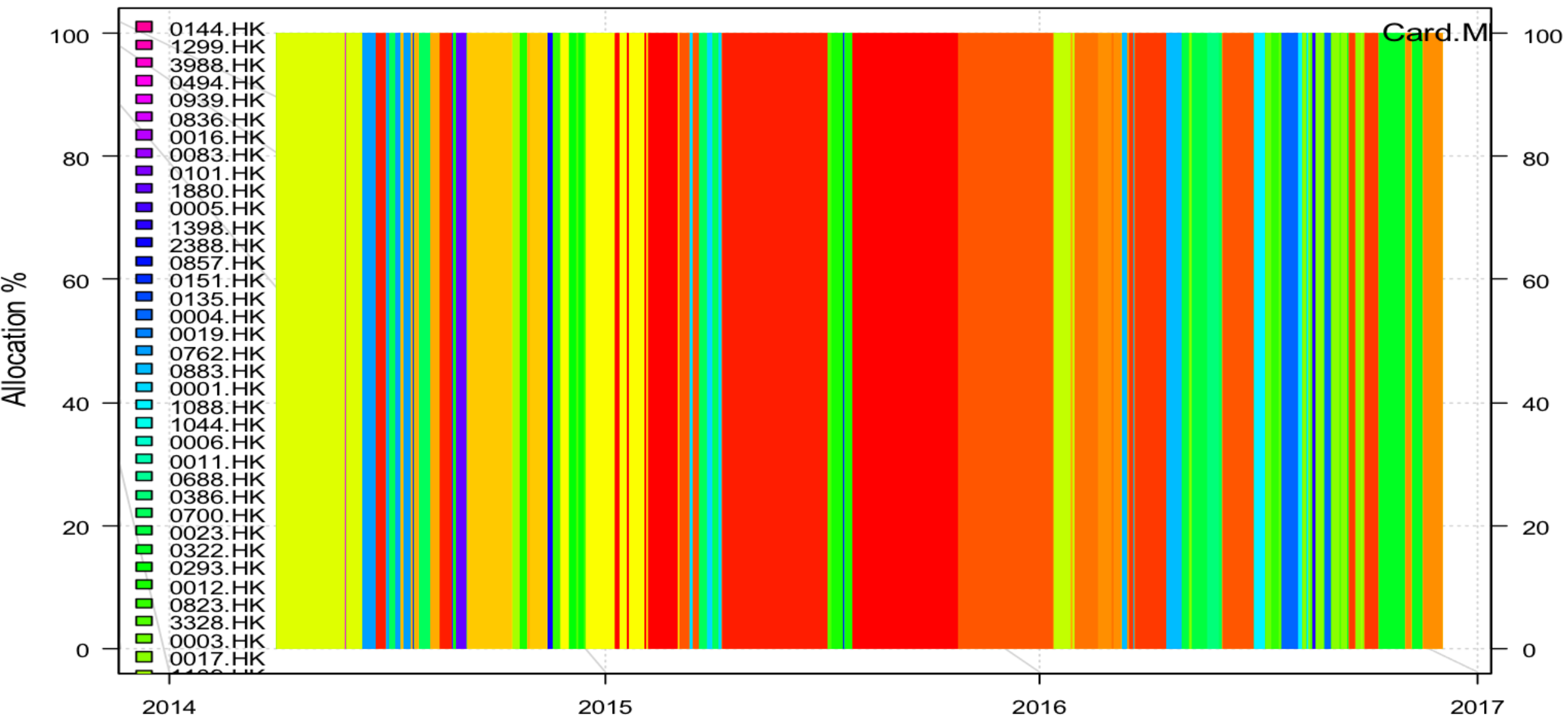
Transition Map for MvLS



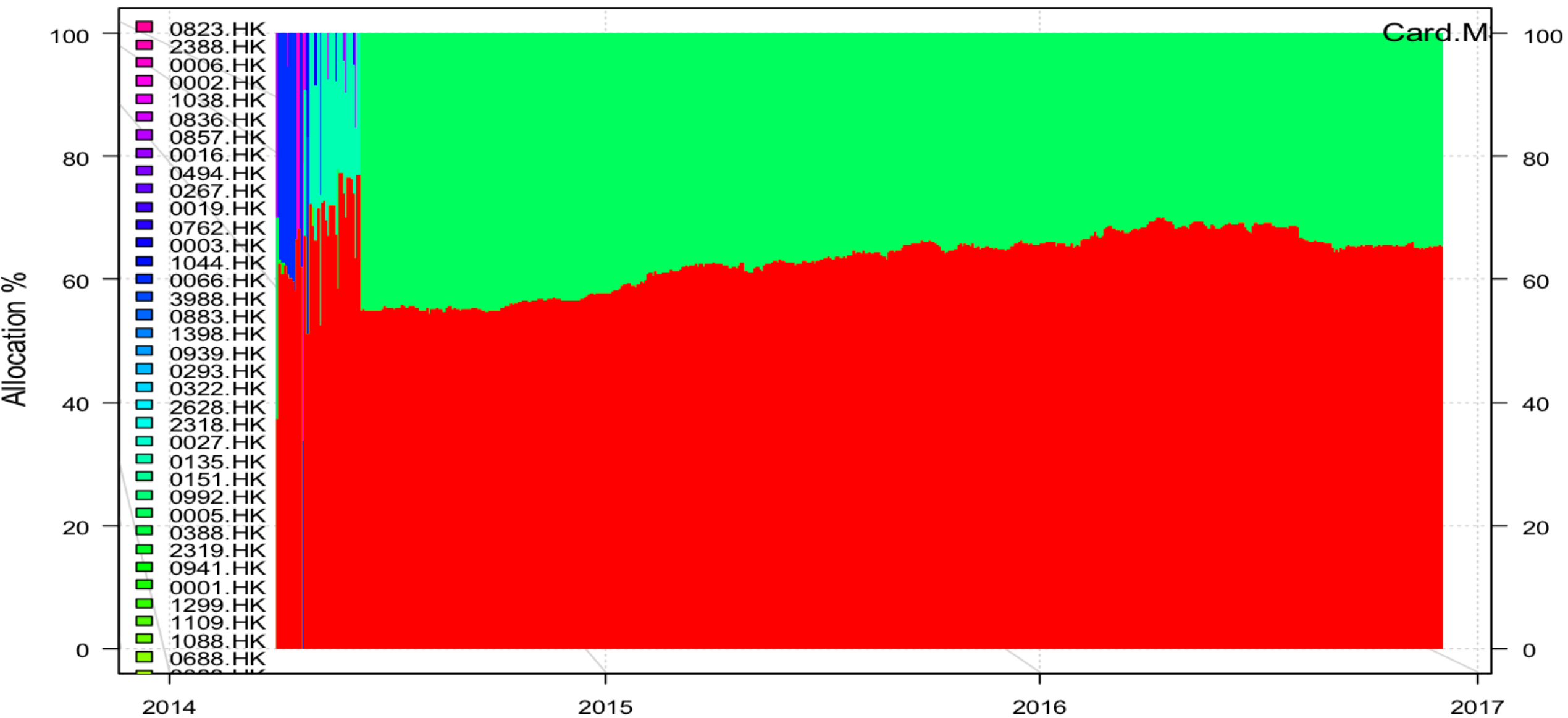
Transition Map for MVL



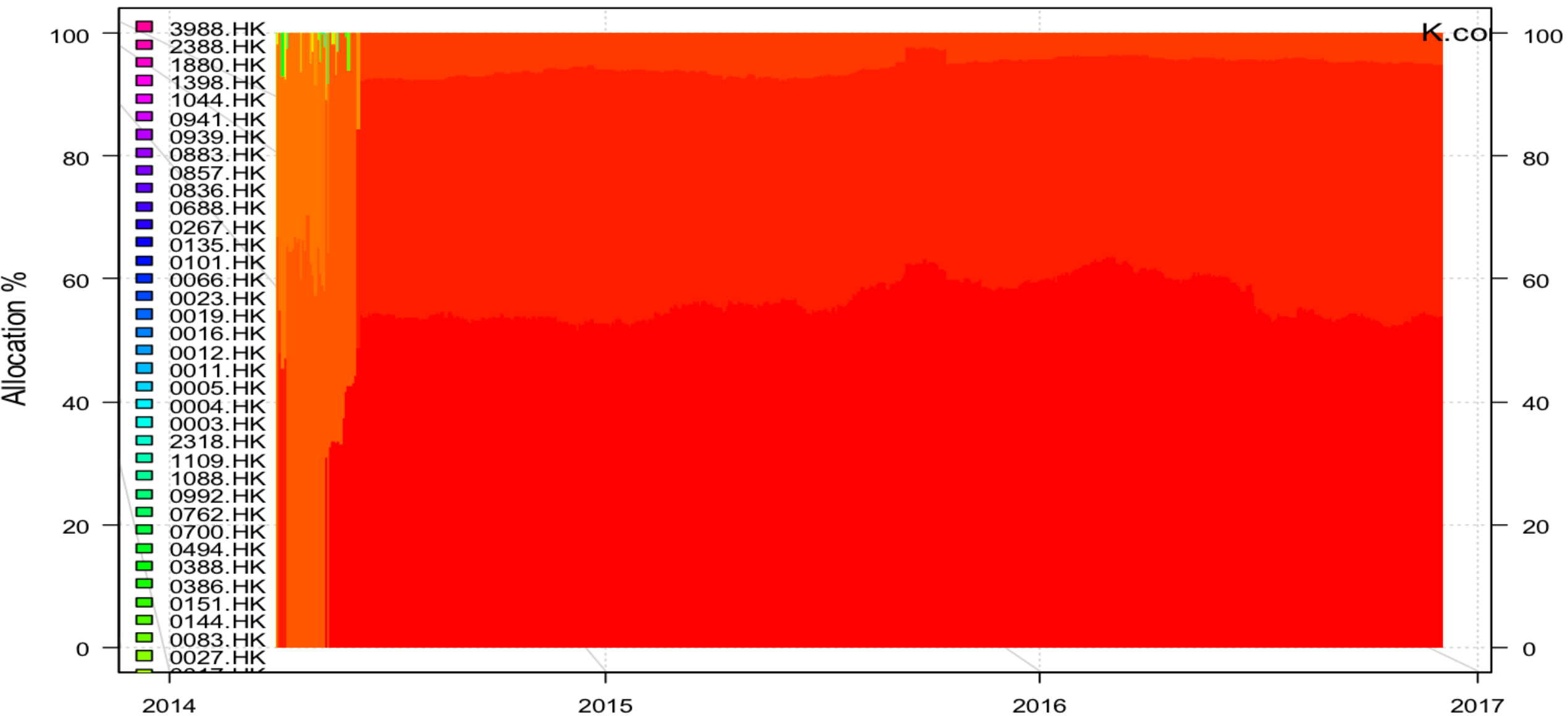
Transition Map for Card.MR



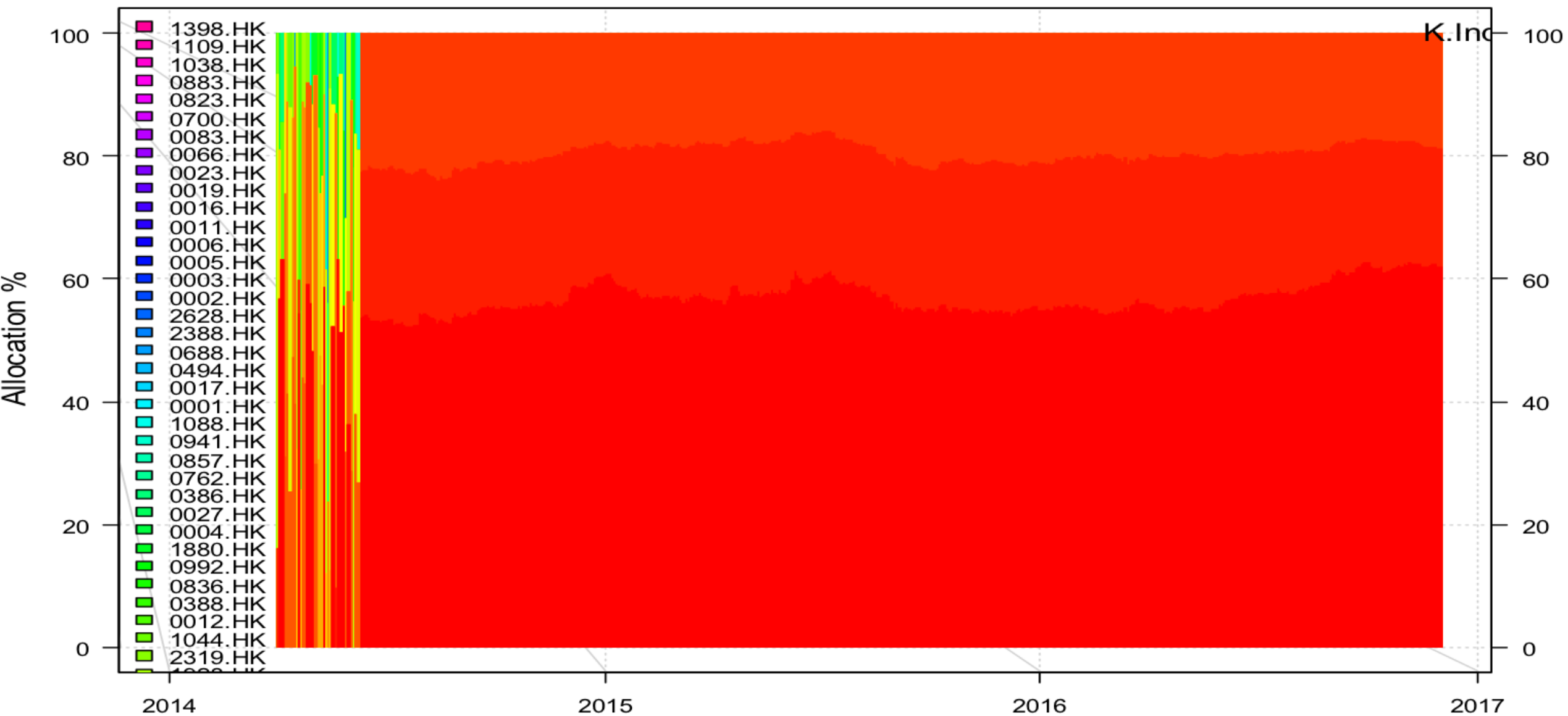
Transition Map for Card.MS



Transition Map for K.cor

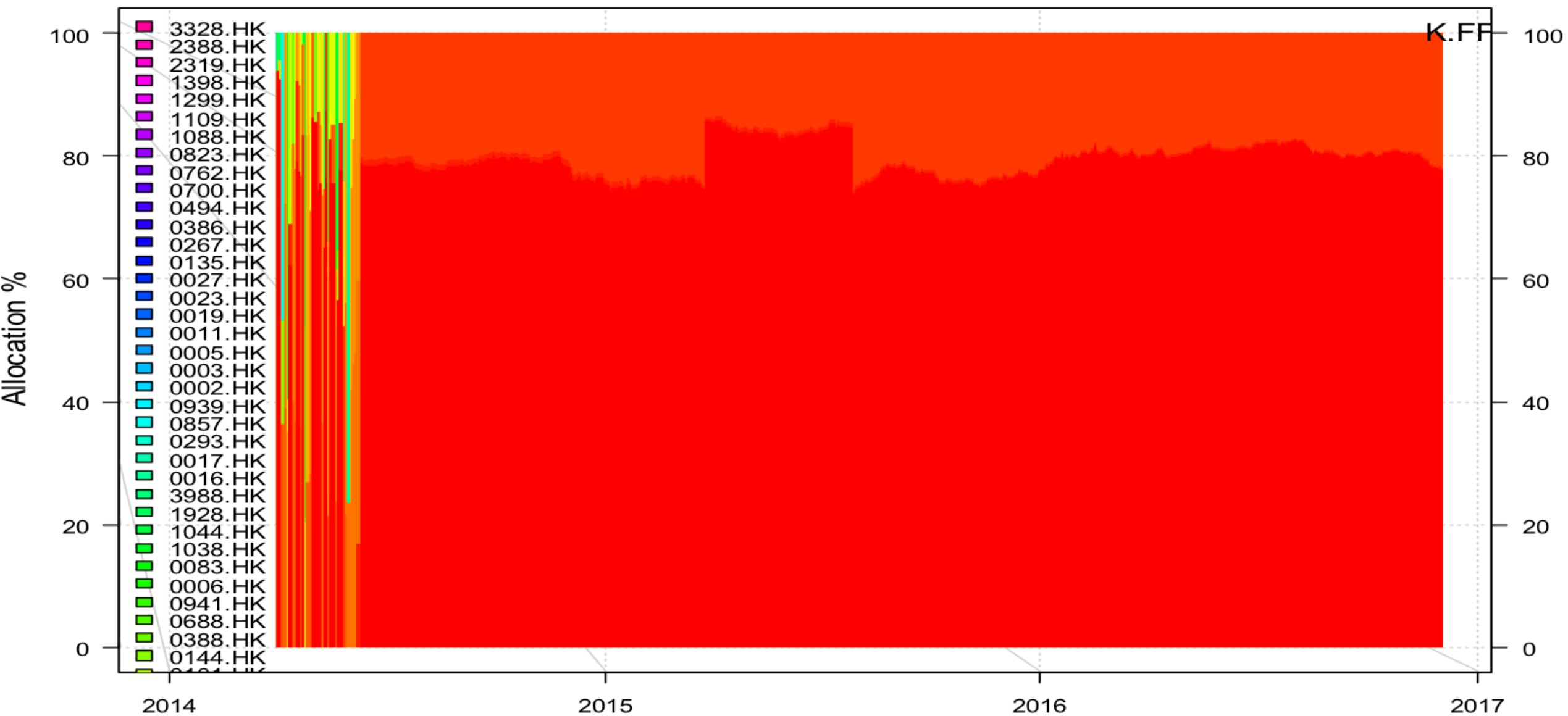


Transition Map for K.Ind

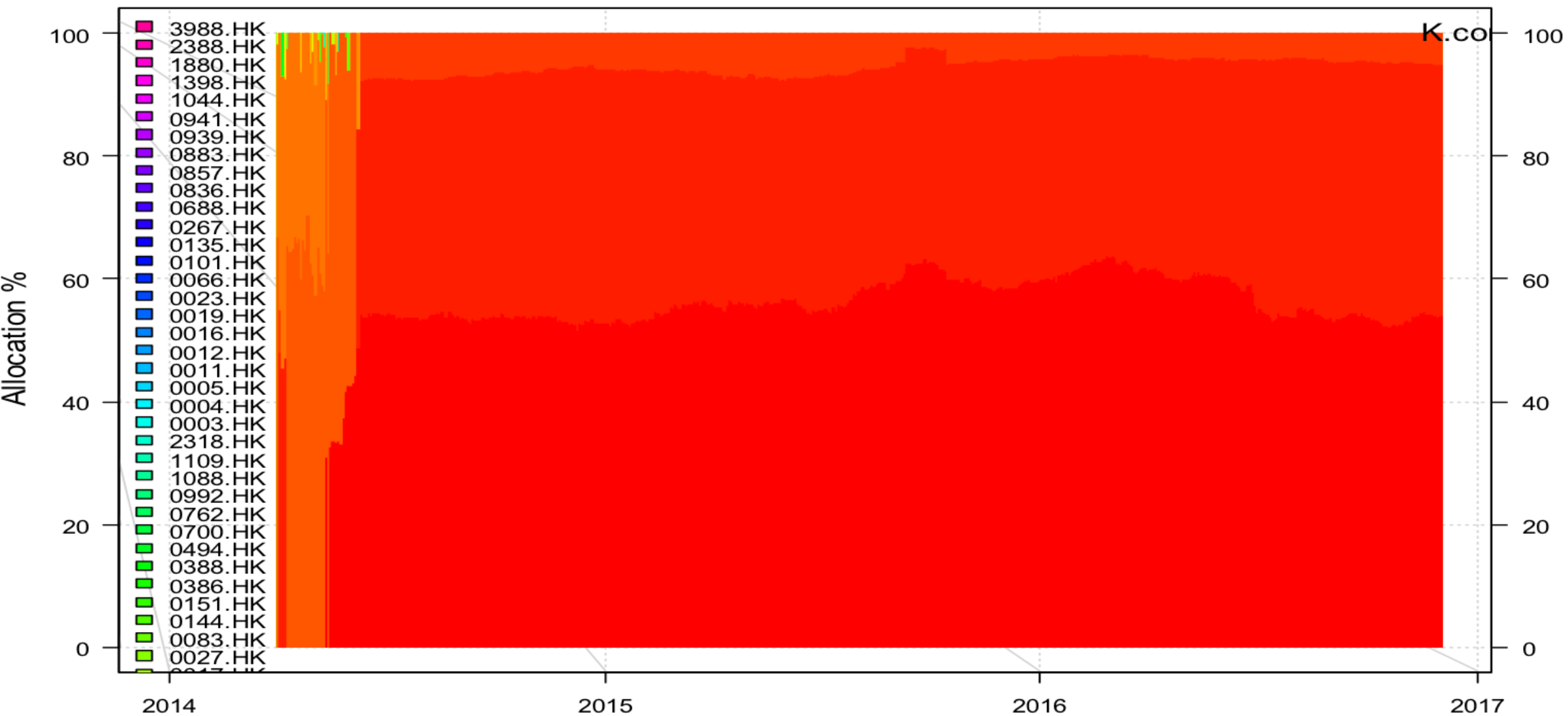




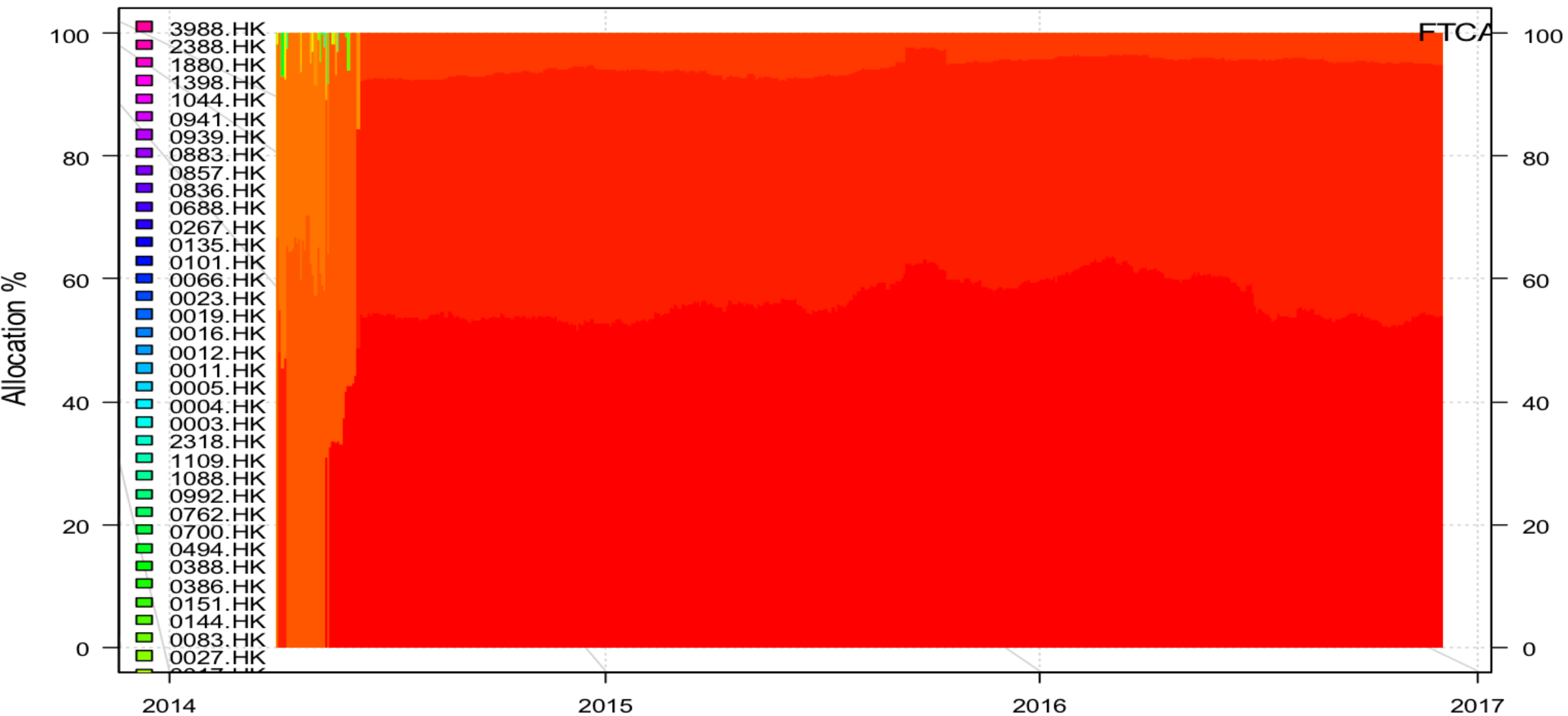
Transition Map for K.FF

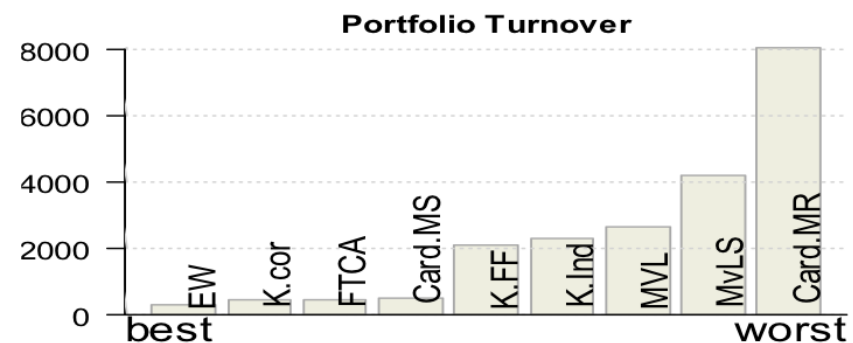
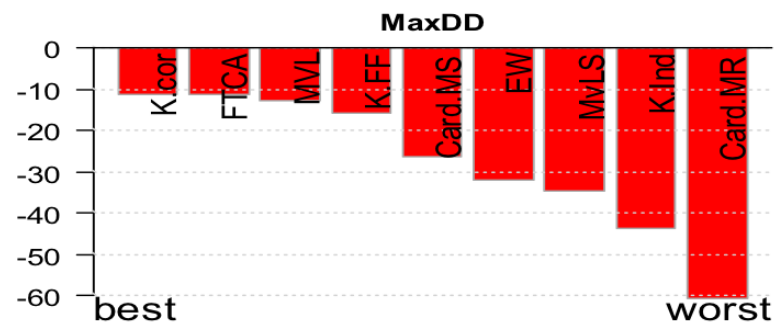
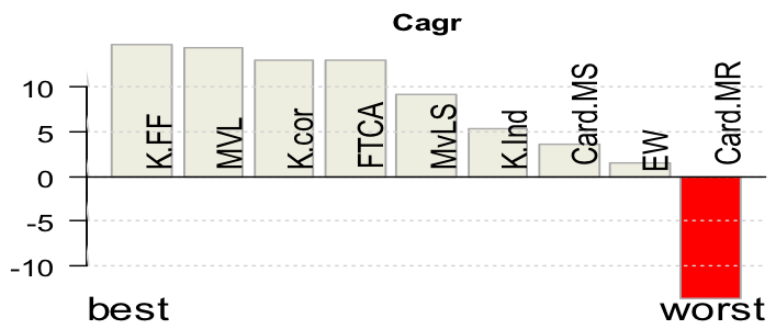
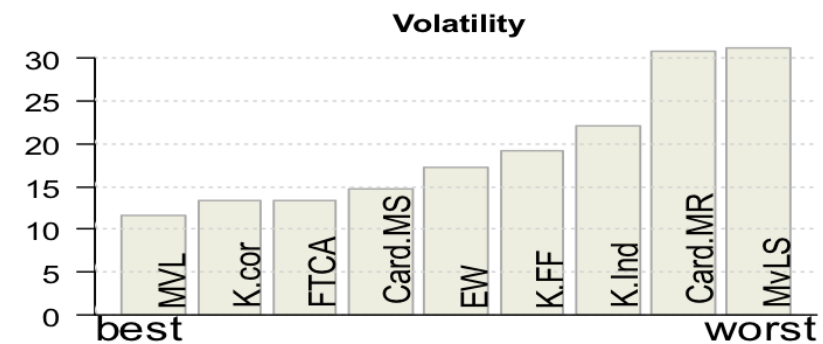
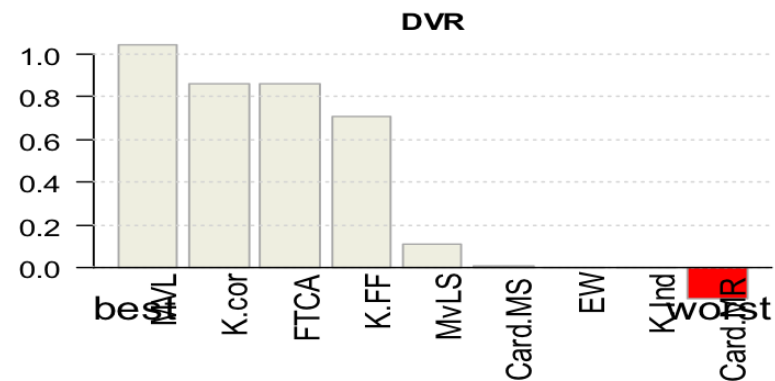
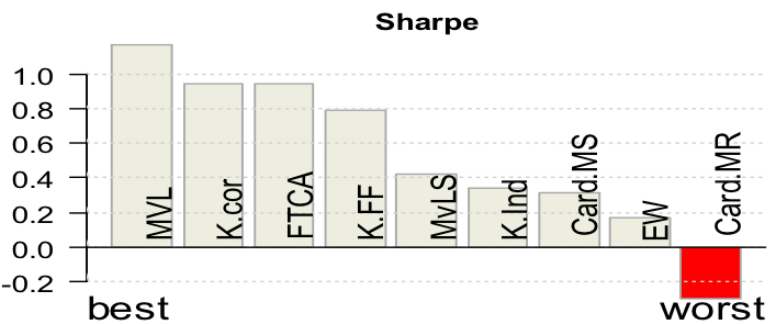


Transition Map for K.cor



Transition Map for FTCA







04

# Review

Future work >>> Project timeline review >>> Q & A



# Future Work

Possible directions



# 01. Problem modification

---

>> Changing Obj.

**MAD**, m.CVaR, M.Sharpe

>> Various constraints

- Shorting allowed
- Adding linear transaction cost constraints
- Minimum/maximum weight constraints



## 02. Factor model

Buffett's model 7 factors

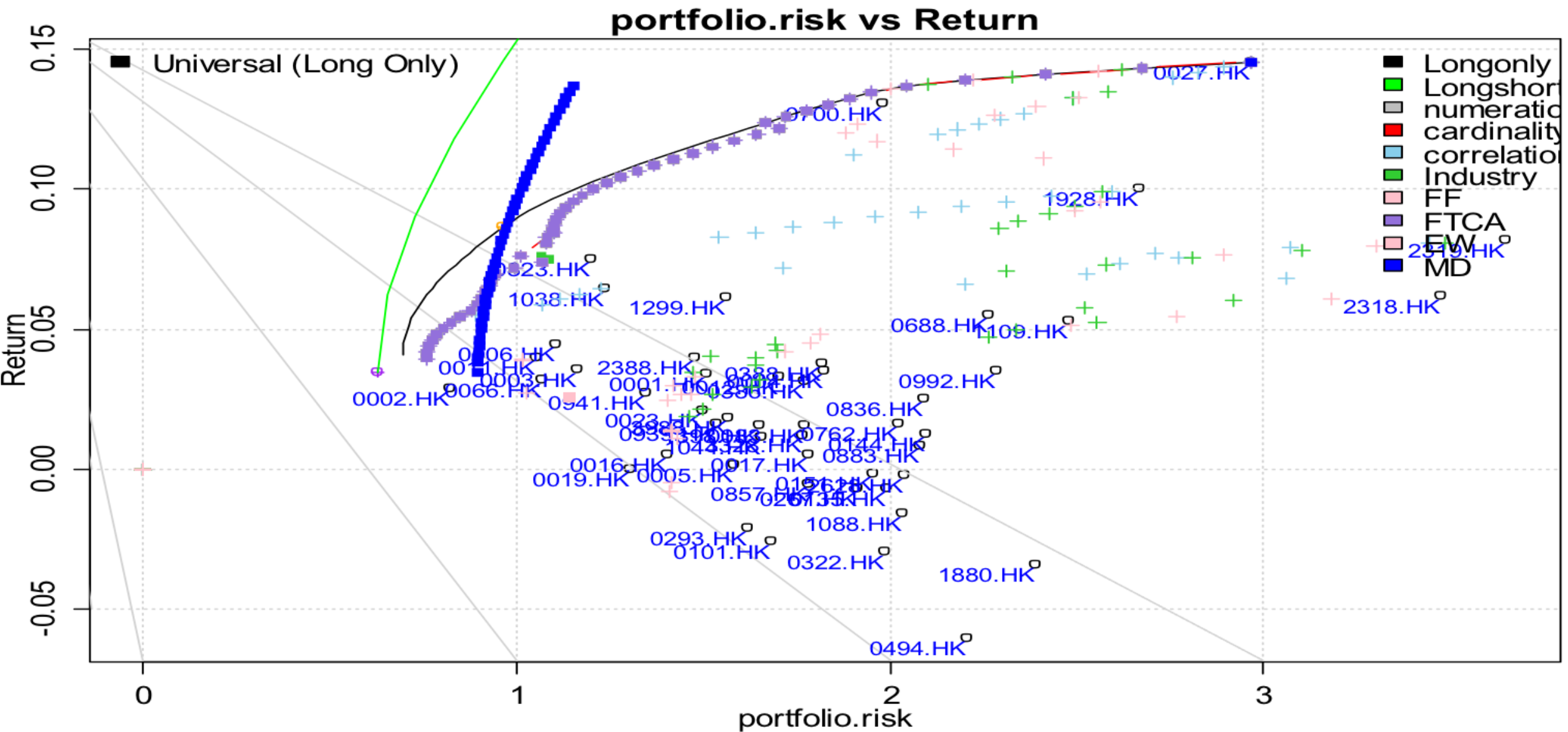
## 03. Clustering

Hierarchical

## 04. Dynamic settings

Portfolio rebalancing

Portfolio turnover







# 01. Problem modification

---

>> Changing Obj.

MAD, m.CVaR, M.Sharpe

>> Various constraints

- Shorting allowed
- Adding linear transaction cost constraints
- Minimum/maximum weight constraints



## 02. Factor model

Buffett's model 7 factors

## 03. Clustering

Hierarchical

## 04. Dynamic settings

Portfolio rebalancing

Portfolio turnover



# 01. Problem modification

---

>> Changing Obj.

MAD, m.CVaR, M.Sharpe

>> Various **constraints**

- **Shorting** allowed
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## 02. Factor model

Buffett's model 7 factors

## 03. Clustering

Hierarchical

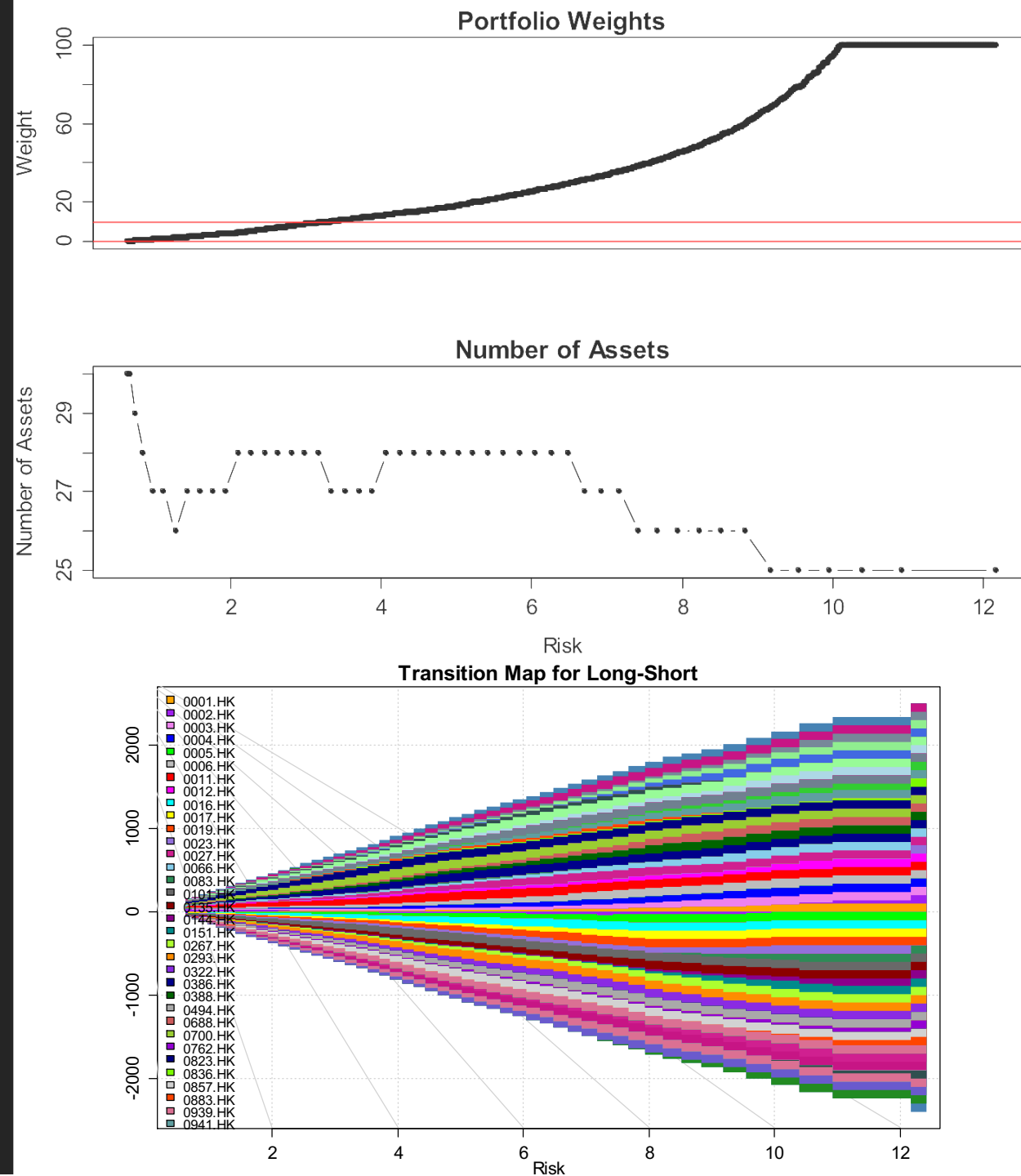
## 04. Dynamic settings

Portfolio rebalancing

Portfolio turnover

# Long & short portfolio

$$\begin{aligned} \min_x & x' Q x \\ \text{s.t. } & \mathbf{r}' x \geq \bar{r} \\ & \mathbf{1}' x = 1 \end{aligned}$$



# CCMV portfolio

$$\min_x x' Q x$$

$$\text{s.t. } r'x \geq \bar{r}$$

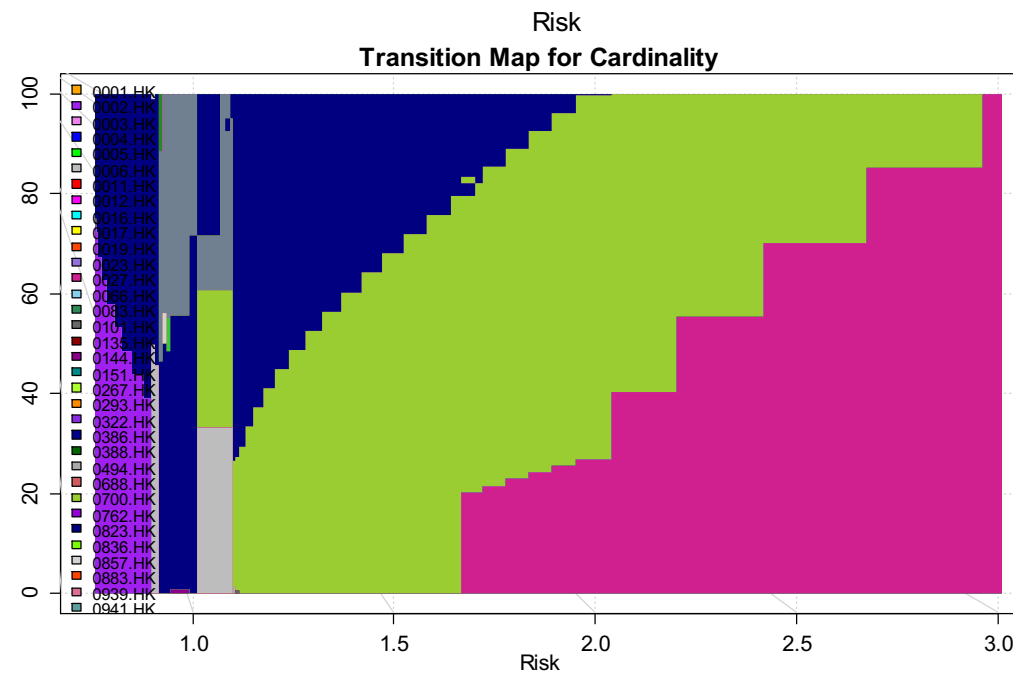
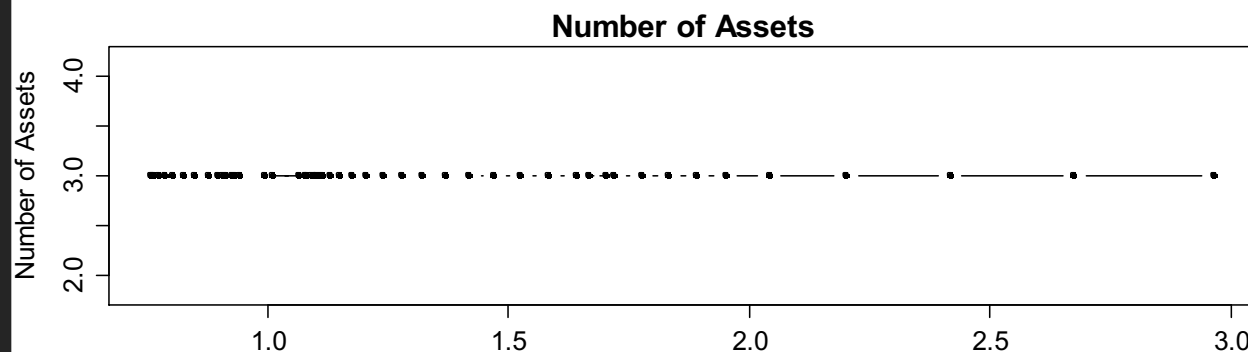
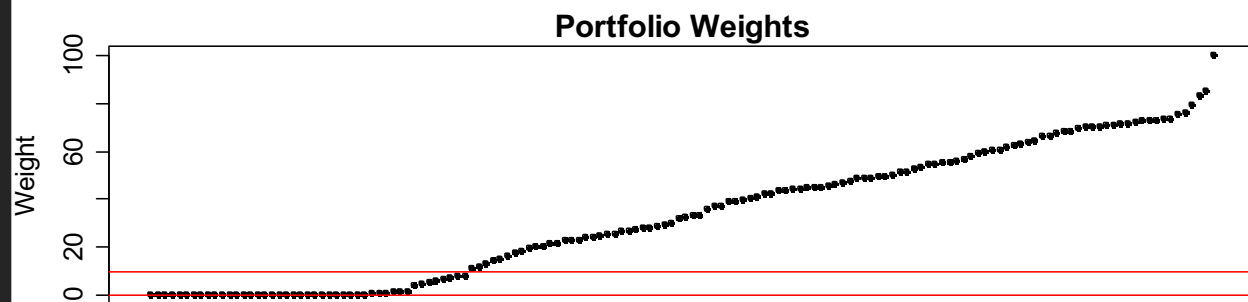
$$\mathbf{1}'x = 1$$

$$0 \leq x_i \leq 1$$

$$\sum_{i=1}^n b_i = k$$

$$b_i = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i = 0 \end{cases}$$

for  $i$  from 1 to  $n$



$k = 3$





# 01. Problem modification

---

>> Changing Obj.

MAD, m.CVaR, M.Sharpe

>> Various constraints

- Shorting allowed
- Adding linear transaction cost constraints
- Minimum/maximum weight constraints



## 02. Factor model

Buffett's model 7 factors

## 03. Clustering

Hierarchical

## 04. Dynamic settings

Portfolio rebalancing

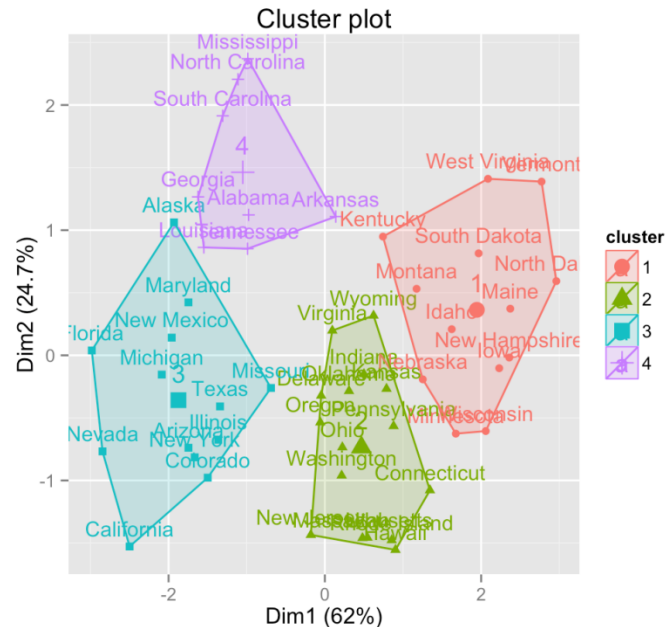
Portfolio turnover

# Step 2: Grouping risky assets using clustering algorithm

**Goal:** partition all risky assets into  $k$  groups risky assets using clustering algorithm

## Partitioned clustering

Organize elements into disjoint groups

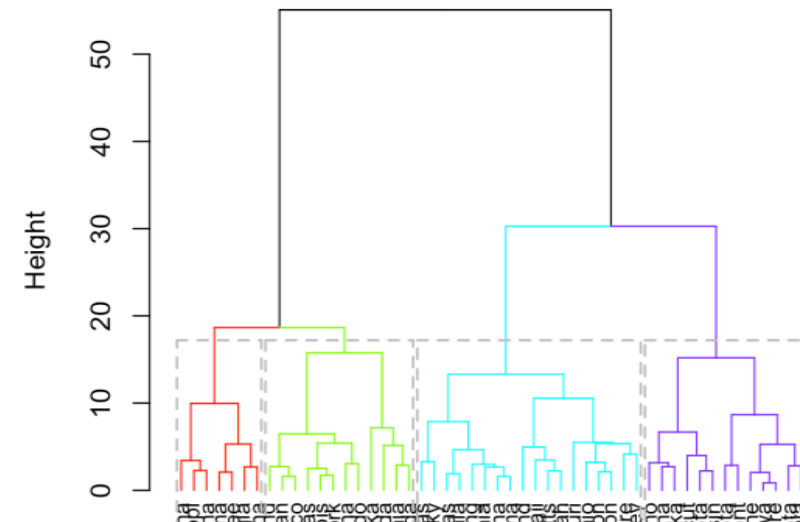


## Hierarchical clustering

Organize elements into trees.

Nested!

### Cluster Dendrogram





# 01. Problem modification

---

>> Changing Obj.

MAD, m.CVaR, M.Sharpe

>> Various constraints

- Shorting allowed
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## 02. Factor model

Buffett's model 7 factors

## 03. Clustering

Hierarchical

## 04. Dynamic settings

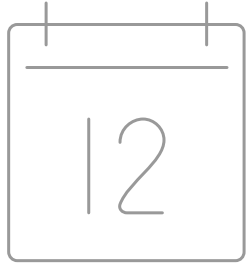
Portfolio rebalancing

Portfolio turnover



## »» Project timeline look back





# TIMELINE

## INFOGRAPHICS

Before July

Preparation done !

1. Identification of Projects
2. Approach to Prof. LI Duan
3. Work Out a research plan
4. Signature of Supervisor
5. Approval from Major Department
6. Submission to Faculty Office
7. Approval by ELITE Stream Director and Associate Dean (Education)
8. Course Registered on CUSIS



Coding & STAT pick up .. Aug.

Start learning R Programming

- though Coursera
- on the job

Register **Statistic** online courses

Literature reading...

Project schedule planning





JOHNS HOPKINS  
UNIVERSITY

09/17/2016

芸秀 周

has successfully completed

**R Programming**

an online non-credit course authorized by Johns Hopkins University and offered  
through Coursera

Jeff Leek, PhD; Roger Peng, PhD; Brian Caffo, PhD  
Department of Biostatistics  
Johns Hopkins Bloomberg School of Public Health

COURSE  
CERTIFICATE



Verify at [coursera.org/verify/PZJCGQZ9XQ9Q](https://coursera.org/verify/PZJCGQZ9XQ9Q)

Coursera has confirmed the identity of this individual and  
their participation in the course.

## Sep. Data prep.

Determine universal asset pool.  
Data collection from Bloomberg  
Data exploration  
Start to understand the literature (gradually...)

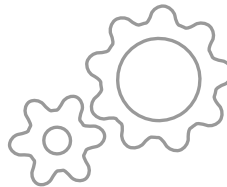
Regular meeting with Prof. Li



## Model realization in R Oct.

Successfully formulated Portfolio  
Optimization problem  
FM.0 Factor model by correlation realized.  
K-means Clustering realized.  
FM.1 Industrial factor model.  
... Debug ...  
..... Debug ...  
..... Debug .....

Constraints formulated!  
Optimization problem.  
Dynamic portfolio building.  
Efficient Frontier explored!



## Since Nov. Polishing up & documentation work

FM.2 Fama-French Model: get FF factors data prep. >>>  
Model development  
Learn about FTCA clustering algorithm >>> FTCA done!



Revision and polishing  
Efficient Frontier revised  
Last week, graduation thesis part One  
Learning about MaD portfolio selection  
Last night, **back testing** and presentation slides ...



# Thanks

Questions and comments are  
welcomed!

Special thanks to my supervisor, **Prof. LI Duan**